

Lattice Calculations and Hadron Physics

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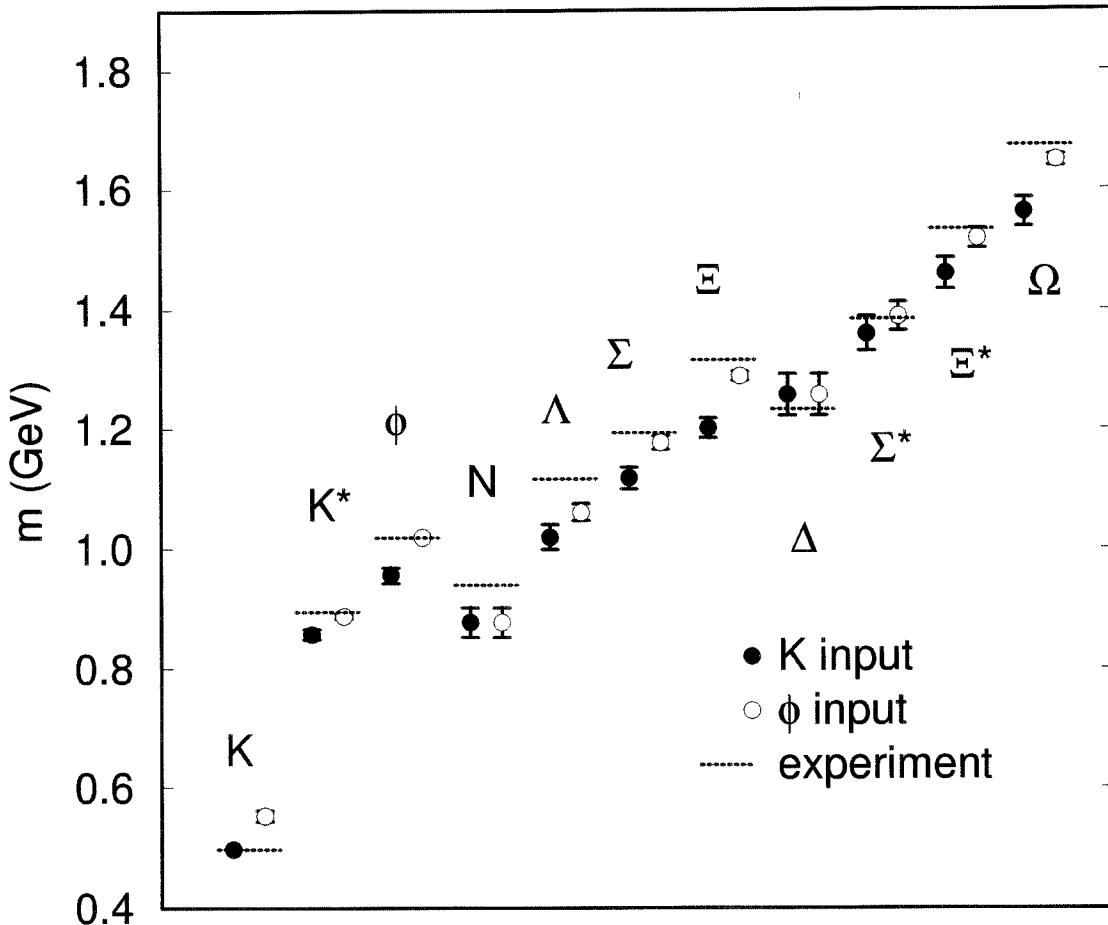
Trends in Lattice QCD calculations

Precise calculations in quenched QCD

neglect quark-antiquark pair creations

Hadron spectra in the continuum limit

CP-PACS98



$K(\phi)$ -input: use $m_K(m_\phi)$ to fix m_s

Quenching errors

- m_{K^*}/m_ϕ (m_K) from $K(\phi)$ input
- strange baryons



Trends move to systematic studies in full QCD

Various attempts for weak matrix elements → CKM



Main issues of the talk

1. CP-violation in K mesons

- strange quark mass
- $K_0-\bar{K}_0$ mixing parameter B_K
- other matrix elements

2. Weak matrix elements of heavy-light mesons

- Leptonic decay constants
- $B_0-\bar{B}_0$ mixing parameter B_B
- Form factors of semi-leptonic decays
 - $D \rightarrow K^{(*)}\ell\nu, \pi(\rho)\ell\nu$
 - $B \rightarrow D^{(*)}\ell\nu$
 - $B \rightarrow \pi(\rho)\ell\nu$

3. Impact on determination of CKM matrix

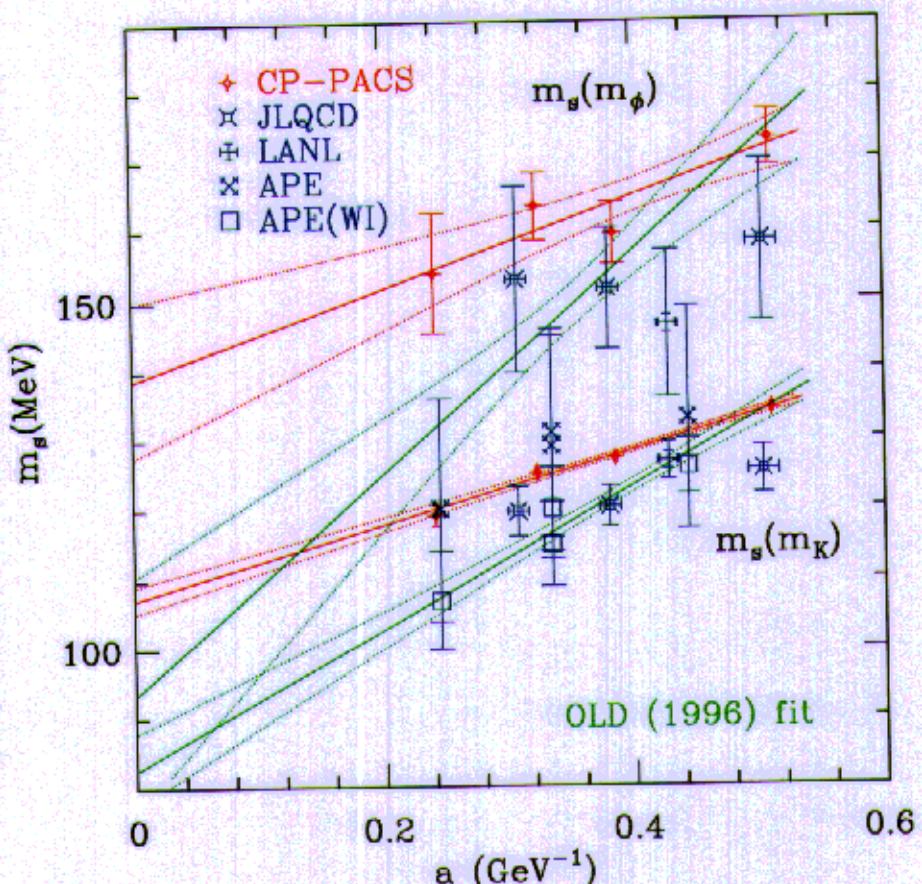
4. Summary and Outlook

1. CP-violation in K mesons

Strange quark mass

Previous lattice effort

compiled by Bhattacharya and Gupta⁹⁷



Quenched result

$\downarrow_{\text{stat.}}$ $\downarrow_{\text{syst.}}$

BG97

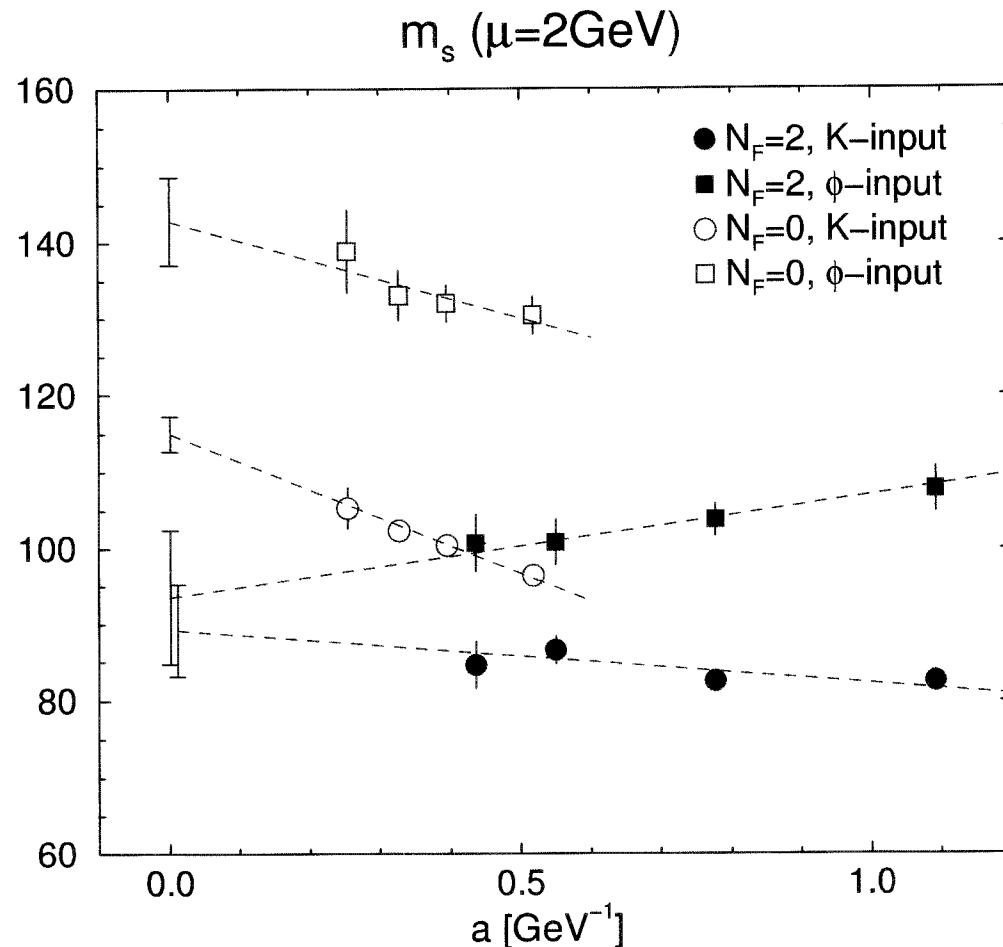
$$m_s(2\text{GeV}) = 110(20)(11) \text{ MeV}$$

sum rule bound: $m_s(2\text{GeV}) \geq 90 - 100 \text{ MeV}$

- ambiguity in a (lattice spacing) $\rightarrow 0$ limit
- K and ϕ inputs disagree (**quenching errors**)

Full QCD

CP-PACS99



N_f	$K\text{-input}$	$\phi\text{-input}$
0	115(2) MeV	143 (6) MeV
2	89.3(6.6)	93.7(8.8)

- m_s in full QCD is smaller than in quenched QCD
- K and ϕ inputs agree \Leftrightarrow correct values for m_{K^*} , m_ϕ and m_K in $a \rightarrow 0$ limit

CP-PACS summary ($N_f = 2$)

$$m_s^{\overline{MS}} (2 \text{ GeV}) = 91 (13) \text{ MeV}$$

$$(m_s^{\overline{MS}} (1.3 \text{ GeV}) = 100 (15) \text{ MeV})$$

Systematics

- $N_f = 2 \rightarrow N_f = 3$: $m_s(N_f = 2) > m_s(N_f = 3)$?
- dynamical quark is not so light ($m_\pi/m_\rho \geq 0.6$)
- renormalization: $m^{\overline{MS}}(\mu) = Z(g^2, \mu a) m^{lat.}(a)$
 Z evaluated at 1-loop is used

Implication $\epsilon'/\epsilon \propto m_s^{-2}$

$Re(\epsilon'/\epsilon) = 21.8(3.0) \times 10^{-4}$ (experiments, world average)

$m_s(m_c) = 100(20)$: this talk		$m_s(m_c) = 130(25)$: Buras(99)	
$m_s(m_c)$	ϵ'/ϵ	$m_s(m_c)$	ϵ'/ϵ
80 MeV	21.2×10^{-4}	105 MeV	11.5×10^{-4}
100	13.0	130	7.0
120	8.5	155	4.4

Other parameters
 $B_6^{1/2} = 1.0$, $B_8^{3/2} = 0.8$, $\Lambda_{\overline{MS}}^{(4)} = 340$ MeV,
 $\text{Im}\lambda_t = 1.33 \cdot 10^{-4}$, $\Omega_{\eta+\eta'} = 0.25$

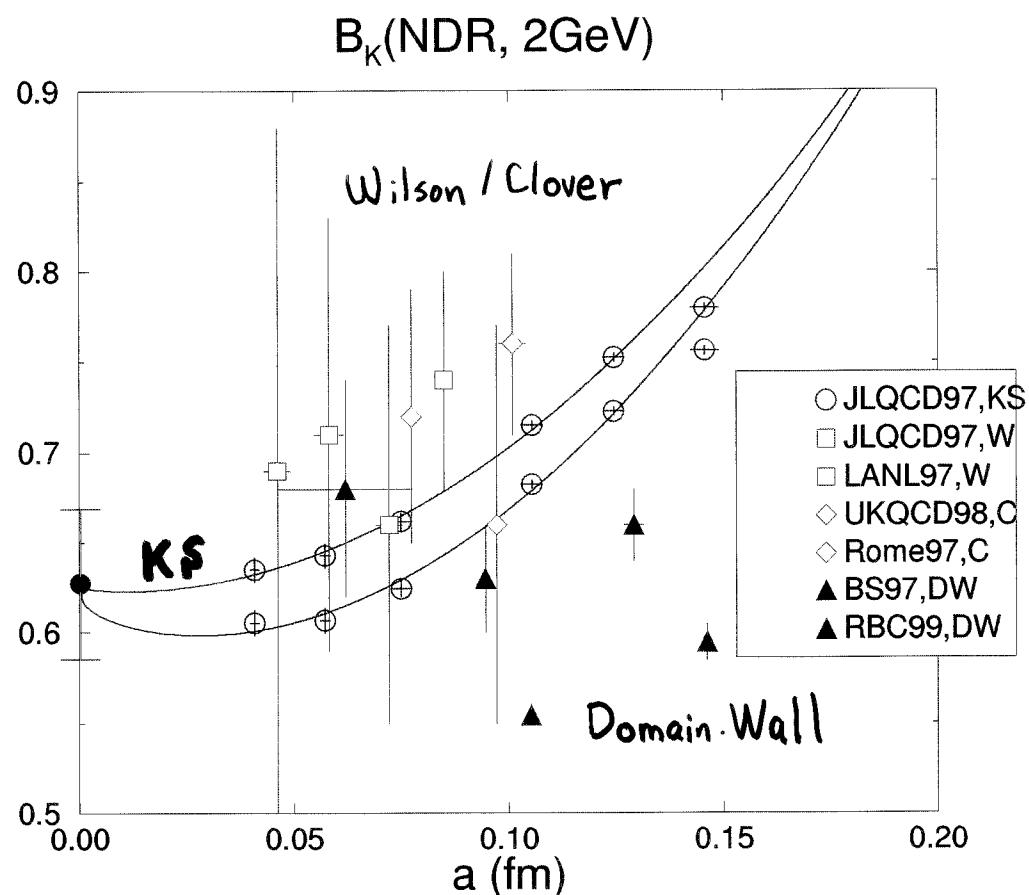
Standard model may not be ruled out by ϵ'/ϵ .

K_0 - \bar{K}_0 mixing parameter B_K

$$\langle \bar{K}_0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A} | K_0 \rangle = \frac{8}{3} B_K(\mu) F_K^2 m_K^2$$

$\epsilon \propto B_K \leftrightarrow \text{CKM matrix}$

Lattice results with different fermion actions



- KS: good chiral behaviour, most accurate
- Wilson/Clover: bad chiral behaviour, Non-Perturbative renormalization needed
- Domain-Wall fermion: good chiral behaviour, preliminary

Result with KS fermions

JLQCD97

$$B_K \text{ (NDR, 2 GeV) } = 0.628 \text{ (42)}$$

$$\text{(RG invariant one: } \hat{B}_K = 0.87(6) \text{)}$$

- 1-loop renormalization factor
- $a \rightarrow 0$ extrapolation with $c_0 + c_1 \cdot a^2 + c_2 \cdot \alpha_s(1/a)^2$
- degenerate quark masses ($m_d = m_s$)
 \Rightarrow CPT predicts $4 \sim 8\%$ increase if $m_d \neq m_s$

Sharpe97

- $\simeq 5\%$ increase in full QCD at $N_f = 3$

Kilcup-Pekurovsky-Venkatamaran97

No chiral/continuum extrapolations, still $m_d = m_s$

Other B parameters, $\Delta I = 1/2$ and ϵ'/ϵ

$\Delta I = 3/2$ EM-penguin: $B_7^{3/2}, B_8^{3/2}$

group	$B_7^{3/2}$	$B_8^{3/2}$	action	a	Renor.
GBS96	$0.58(2)(\frac{7}{3})$	$0.81(3)(\frac{3}{2})$	W	$\neq 0$	Pert.
GKS97	$0.62(3)(6)$	$0.77(4)(4)$	KS	$\rightarrow 0$	Pert.
Rome97	$0.72(5)$	$1.03(3)$	C	$\neq 0$	non-Pert.
Rome97	$0.58(2)$	$0.83(2)$	C	$\neq 0$	Pert.

$\Delta I = 1/2, \epsilon'/\epsilon$

$K \rightarrow \pi\pi$ amplitude: notoriously difficult

$\Downarrow \leftarrow$ chiral perturbation theory

$K \rightarrow \pi$ amplitude

- some subtraction is needed
- chiral symmetry for fermion action is crucial

KS: GKS90, Pekurovsky-Kilcup98

DWF: RBC99

Results

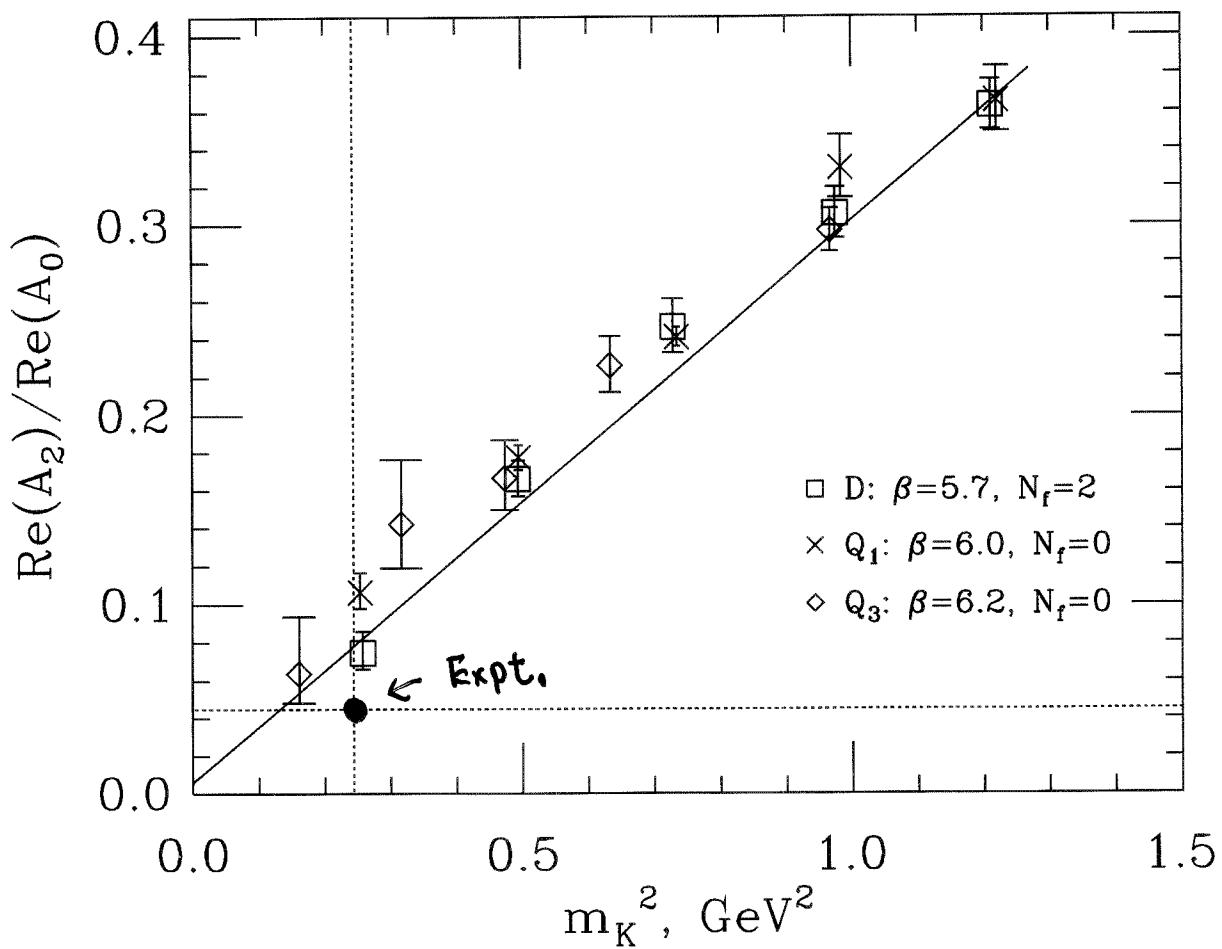
KS DW
↑ ↑

- signal for $\text{Re}(A_2)/\text{Re}(A_0)$ PK98, RBC99
- signal for $B_6^{1/2}$ / huge matching errors PK98

need more work for definite conclusions

$\text{Re}(A_2)/\text{Re}(A_0)$

Pekurovsky-Kilcup98



2. Weak matrix elements of heavy-light mesons

Problem of heavy quark on the lattice

- $a^{-1} = 1 \sim 4$ GeV in currents simulations
- $m_b \simeq 4$ GeV larger than a^{-1} (cut-off)

Recent developments

The following 3 methods, employed complementarily

1. Charm \oplus Static ($m_Q = \infty$) $\rightarrow m_b$ (Extrapolation)
2. Non-Relativistic QCD (NRQCD)
3. Simulation on $m_b a \oplus$ non-relativistic interpretation (Direct)

Method	$a \rightarrow 0$ limit	m_b	m_c
Extra.	simple	not direct	OK
NRQCD	does not exist	direct	difficult
Direct	complicated	direct	OK

Agreements among 3 methods

→ measure of reliability

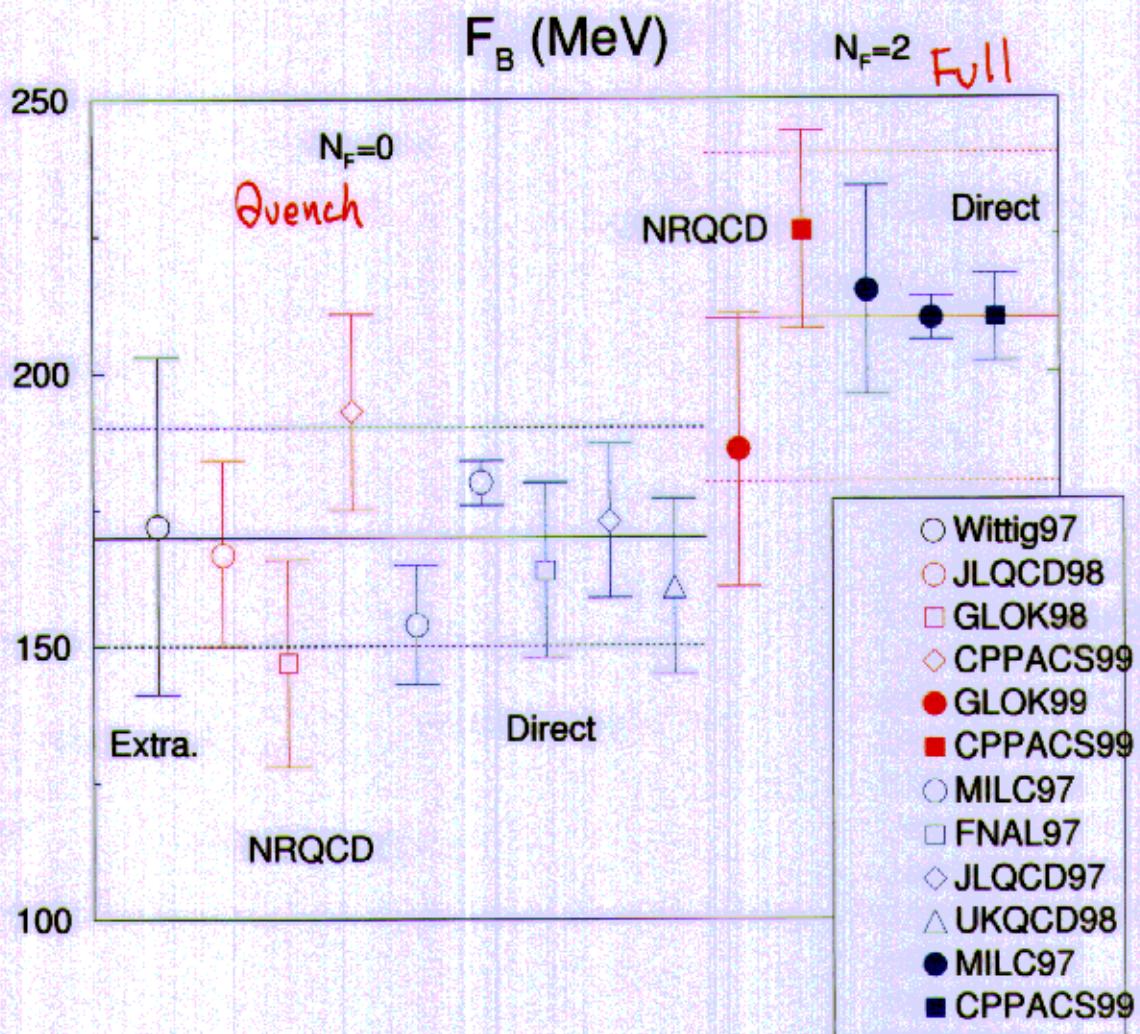
Leptonic Decay constants

Quenched QCD

Results from above 3 methods are consistent

Full QCD

effect seems large $\rightarrow 10 \sim 20\%$ increase of F_B



Summary of lattice results

	$N_f = 0$	$N_f = 2$	Expt.
F_B	$170(20)$ MeV	$210(30)$ MeV	
F_{B_s}	$195(20)$	$245(30)$	
F_D	$200(20)$		< 310 MeV
F_{D_s}	$220(20)$		$241(21)(30)$
F_{B_s}/F_B	$1.15(4)$	$1.16(4)$	
F_{D_s}/F_D	$1.10(6)$		

Present best estimate

$$F_B = 210 \text{ (30) MeV } (N_f = 2)$$

$B - \bar{B}$ mixing parameter B_B

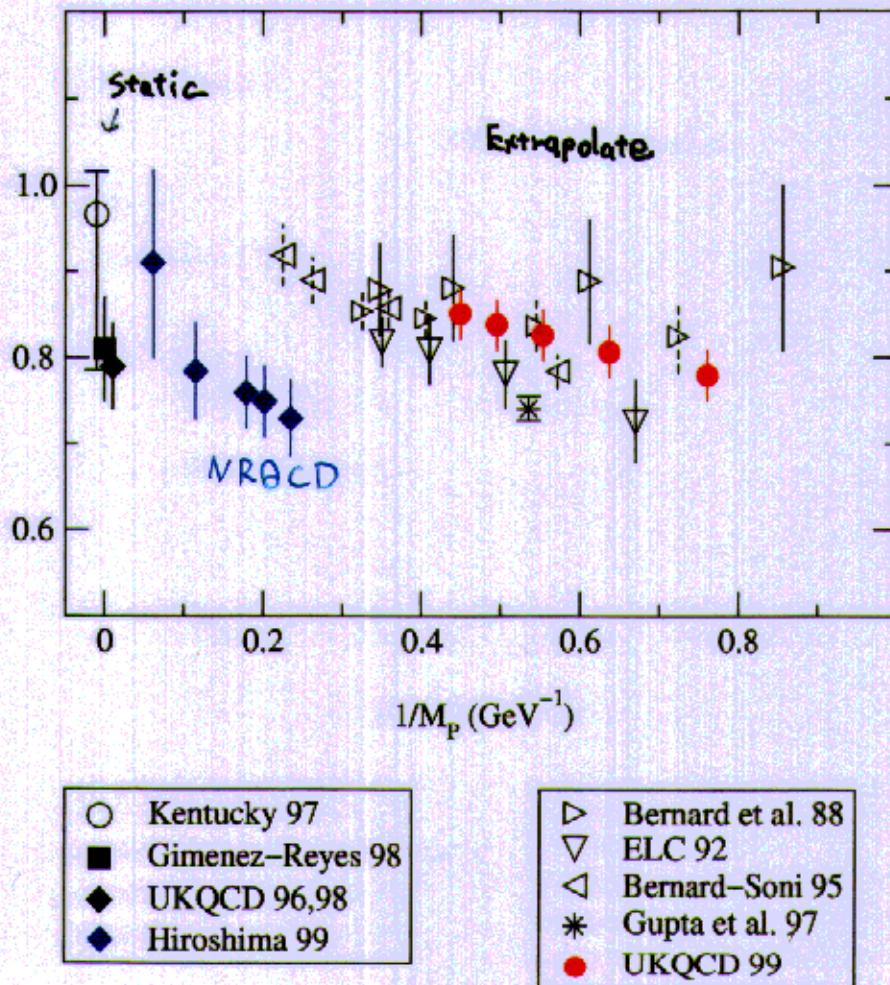
$$\langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B_q \rangle = \frac{8}{3} B_{B_q}(\mu) F_{B_q}^2 m_{B_q}^2$$

where $q = d, s$.

$$\Delta M_q = \frac{G_F^2 M_W^2}{6\pi^2} \eta_{B_q} S(m_t/M_W) \textcolor{red}{F}_{B_q}^2 \hat{B}_{B_q} |V_{tq}|^2$$

Mass dependence

Quenched approximation, $a \neq 0$



1. Static-Light Kentucky97, Gimenez-Reyes98, UKQCD96,98

$$B_{B_d}(m_b) = 0.80(5) \quad (\hat{B}_{B_d} = 1.28(8))$$

$$\xi_{sd}^2 \equiv \frac{F_{B_s}^2 B_{B_s}}{F_{B_d}^2 B_{B_d}} = 1.38(15)$$

2. NRQCD-Light Hiroshima99

mass dependence is observed

$$B_{B_d}(m_b) = 0.75(3)(12)$$

resonably consistent with Static in the static limit

3. Extrapolation from charm/direct

Bernard et al.88, ELC92, BS95, Gupta et al.97, UKQCD99

seems to disagree with Static in the static limit

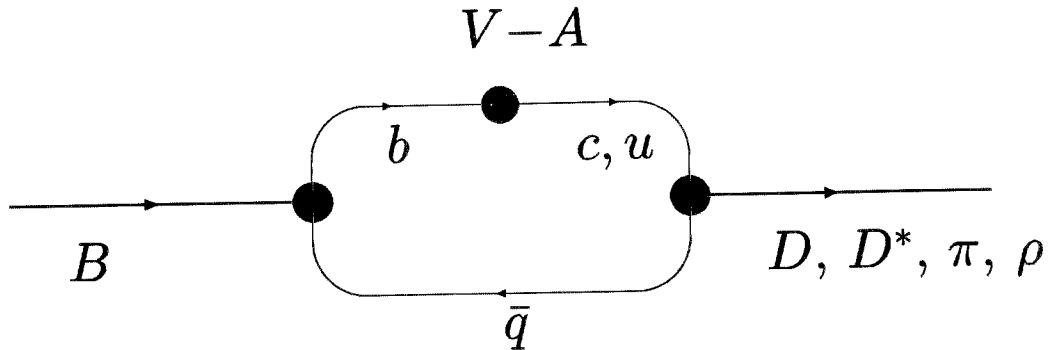
need more work to have consistent results

Tentative conclusion (Static result)

$$B_{B_d}(m_b) = 0.80(5) \quad (\hat{B}_{B_d} = 1.28(8))$$

$$\xi = 1.17(6), \quad \sqrt{\hat{B}_B} F_B = 240 \text{ (36) MeV}$$

Form factors of semi-leptonic decays



PseudoScalar → PseudoScalar

$$\langle P(k)|V^\mu|H(p)\rangle = f^+(q^2) \left[(p+k)^\mu - \frac{m_H^2 - m_P^2}{q^2} q^\mu \right] + f^0(q^2) \frac{m_H^2 - m_P^2}{q^2} q^\mu$$

where $q = p - k$ is the momentum transfer, $H = B$ or D , and $P = D, K, \eta$ or π .

PseudoScalar → Vector

$$\langle V(k, \varepsilon)|V^\mu|H(p)\rangle = \frac{2V(q^2)}{m_H + m_V} \epsilon^{\mu\nu\alpha\beta} p_\nu k_\alpha \varepsilon_\beta^*$$

$$\begin{aligned} \langle V(k, \varepsilon)|A^\mu|H(p)\rangle &= i(m_H + m_V) A_1(q^2) \varepsilon^{*\mu} - i \frac{A_2(q^2)}{m_H + m_V} \varepsilon^* \cdot p (p + k)^\mu \\ &\quad + i \frac{A(q^2)}{q^2} 2m_V \varepsilon^* \cdot p q^\mu \end{aligned}$$

where $V = D^*, K^*, \phi$ or ρ .

Status of form factor calculations

- quenched calculations only (no full QCD)
- no continuum extrapolation

1. $D \rightarrow K^{(*)}\ell\nu, \pi(\rho)\ell\nu$

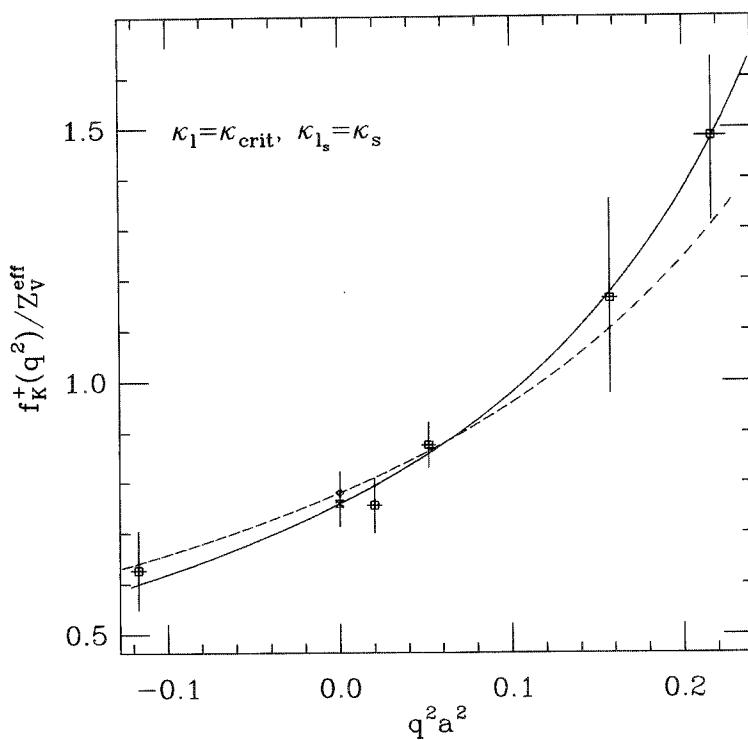
- check of lattice calculations assuming $V_{cs} = 1 - \lambda^2/2$
- determination of V_{cs} (V_{cd})

Easier than other form factors

- direct simulations at m_c / m_s are possible
- all physical q^2 regions are covered
 \Rightarrow interpolate to $q^2 = 0$

$f_+(q^2)$ from $D \rightarrow K\ell\nu$

UKQCD95



Lattice methods are well-established

Summary (Quenched approximation, $a \neq 0$)

WUP97, LANL96, UKQCD95, APE95, ELC94, BKS92, LMMS92

	$D \rightarrow K, K^*$		$D \rightarrow \pi, \rho$
	lattice	expt.	lattice
$f^+(0)$	0.73(7)	0.76(3)	0.65
$V(0)$	1.2(2)	1.07(9)	1.1(2)
$A_1(0)$	0.70(7)	0.58(3)	0.65(7)
$A_2(0)$	0.6(1)	0.41(5)	0.55(10)

- $f^+(0)$ and $V(0)$ agree with experimental values
- $A_1(0)$ and $A_2(0)$ are slightly higher
 \Leftarrow axial-vector current normalization

2. $B \rightarrow D^{(*)}\ell\nu$

large branching ratios
 \Rightarrow precise determination of $|V_{cb}|$

Differential decay rates

$$\frac{d\Gamma}{d\omega}(B \rightarrow D\ell\nu) = (\text{known factor})|V_{cb}|^2|\mathcal{F}_{B \rightarrow D}(\omega)|^2$$

$$\frac{d\Gamma}{d\omega}(B \rightarrow D^*\ell\nu) = (\text{known factor})|V_{cb}|^2|\mathcal{F}_{B \rightarrow D^*}(\omega)|^2$$

$\omega = v_B \cdot v_{D^{(*)}}$: the velocity transfer.

$B \rightarrow D^*\ell\nu$ experiments

Groups	$ \mathcal{F}_{B \rightarrow D^*}(1)V_{cb} (\times 10^{-3})$
ALEPH(97)	$31.9 \pm 1.8 \pm 1.9$
DELPHI(99)	$37.95 \pm 1.34 \pm 1.59$
OPAL(96)	$32.8 \pm 1.9 \pm 2.2$
CLEO(95)	$35.1 \pm 1.9 \pm 1.8$

Contribution of lattice QCD

- the values of $\mathcal{F}_{B \rightarrow D^{(*)}}(1)$ to determine $|V_{cb}|$
- the shape of $\mathcal{F}_{B \rightarrow D^{(*)}}(\omega)$ to extract $|\mathcal{F}_{B \rightarrow D^{(*)}}(1)V_{cb}|$ from experimental data

NB: Heavy Quark Symmetry relation

$$\mathcal{F}_{B \rightarrow D}(1) = \mathcal{F}_{B \rightarrow D^*}(1) = 1$$

for $m_b, m_c \rightarrow \infty$

(1) Determination of $\mathcal{F}_{B \rightarrow D^{(*)}}(1)$

$$\mathcal{F}_{B \rightarrow D}(1) = h_+(1) - \frac{m_B - m_D}{m_B + m_D} h_{A_1}(1)$$

$$\mathcal{F}_{B \rightarrow D^*}(1) = h_{A_1}(1)$$

where

$$\begin{aligned} \frac{\langle D(v') | \bar{c} \gamma_\mu b | B(v) \rangle}{\sqrt{m_B m_D}} &= [h_+(\omega)(v + v')_\mu + h_-(\omega)(v - v')_\mu] \\ \frac{\langle D^*(v') | \bar{c} \gamma_\mu \gamma_5 b | B(v) \rangle}{\sqrt{m_B m_D^*}} &= (\omega + 1) \varepsilon_\mu^* h_{A_1}(\omega) - \dots \end{aligned}$$

Lattice calculation

FNAL99

- quenched approximation, $a^{-1} \sim 1$ GeV
- ratio method \Rightarrow cancellation of errors

$$|h_+^{B \rightarrow D}(1)|^2 = \frac{\langle D | V_0^{cb} | B \rangle \langle B | V_0^{bc} | D \rangle}{\langle D | V_0^{cc} | D \rangle \langle B | V_0^{bb} | B \rangle}$$

- $1/m_{b,c}^3 (1/m_{b,c}^2)$ corrections to $h_+(1)$ ($h_{-,A_1}(1)$)

Results

	Lattice	QCD sum rule
$\mathcal{F}_{B \rightarrow D}(1)$	1.058 (16) ($^{+14}_{-6}$)	0.98(7)
$\mathcal{F}_{B \rightarrow D^*}(1)$	0.935 (22) ($^{+23}_{-24}$)	0.91(6)/0.92(4)

- smaller a or $a \rightarrow 0$ limit
- full QCD effect

(2) Shape of $\mathcal{F}_{B \rightarrow D^{(*)}}(\omega)$

HQS implies

$$\begin{aligned} h_{+,A_1}(\omega) &= (1 + \gamma_{+,A_1}(\omega))\xi(\omega) \\ h_{-}(\omega) &= \gamma_{-}(\omega)\xi(\omega) \end{aligned}$$

$\xi(\omega)$: Isgur-Wise function, mass independent

Current status

$\xi(\omega)$ has already been calculated

Ex. UKQCD99

Next step

extract mass dependent terms $\gamma_{\pm,A_1}(\omega)$

$\Rightarrow h_{\pm,A_1} \Rightarrow \mathcal{F}_{B \rightarrow D^{(*)}}(\omega)$

DEL PHI (99)

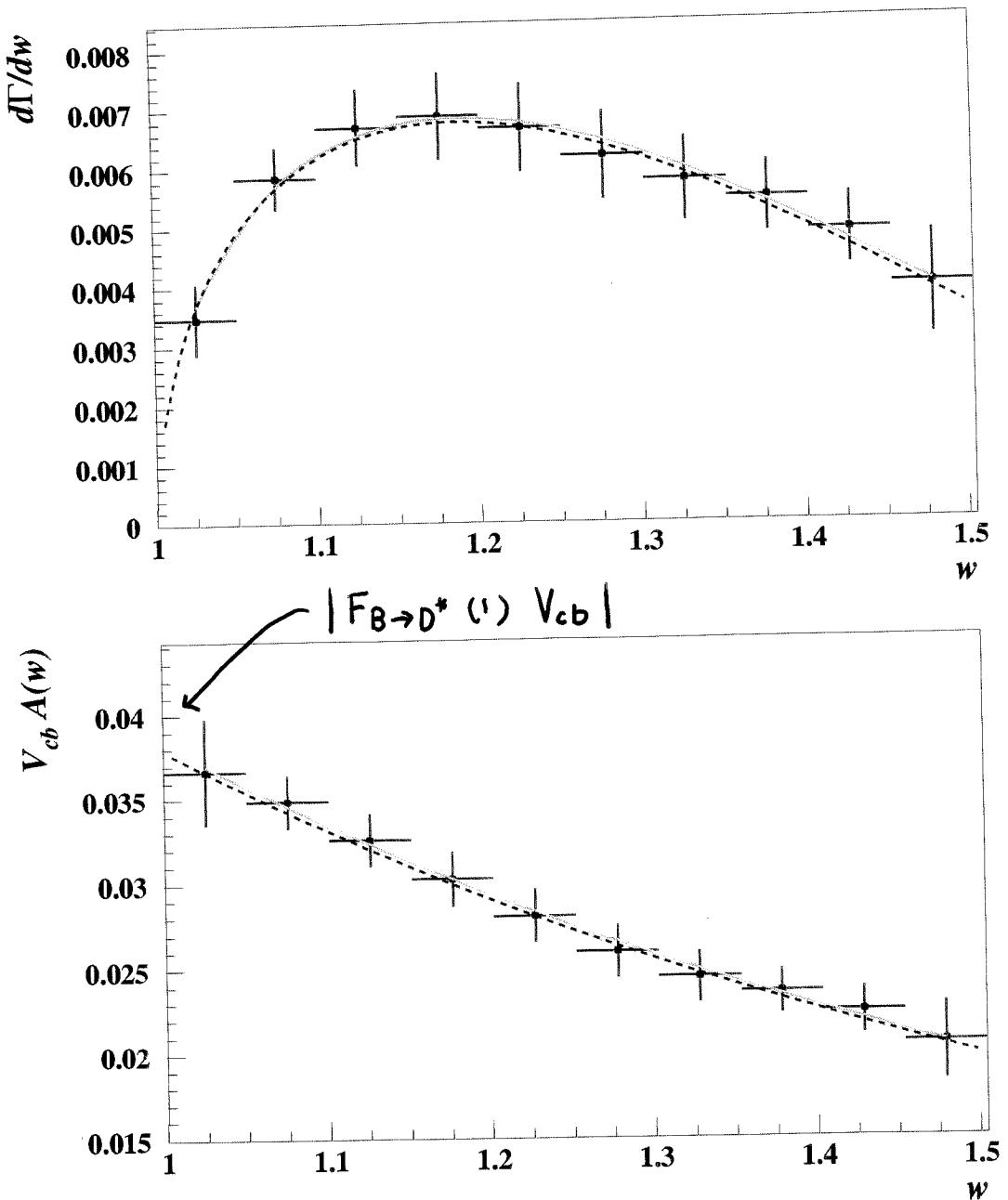
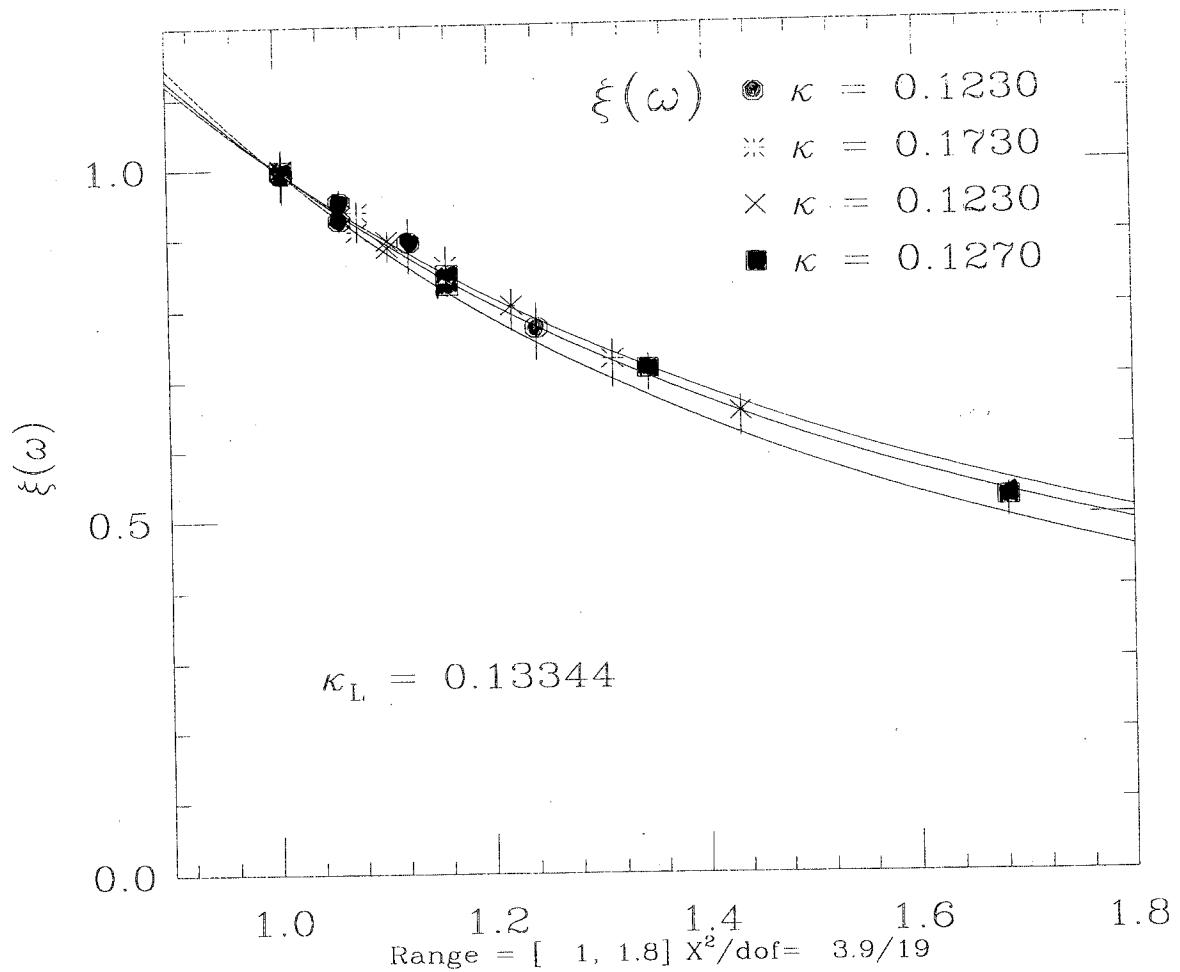


Figure 5: Unfolded distributions. Upper plot: differential decay width. Lower plot: decay form factor as in eq. (8). The continuous lines show the results of a fit to the histograms, neglecting bin to bin correlations. The dotted lines show the result obtained when including the statistical correlations among the bins.

$\xi(\omega)$

UK QCD 99



Slope of Isgur-Wise function Model dependent

$$\xi(\omega) = 1 - \rho^2(\omega - 1) + \dots = \frac{2}{\omega + 1} \exp[-2(\rho^2 - 1)\frac{\omega - 1}{\omega + 1}]$$

Group(Yr)	$\rho_{u,d}^2$	ρ_s^2	Remark
BSS(93)		1.4(4)	$D \rightarrow D$
UKQCD(95)	0.9(4)	1.2($\frac{3}{2}$)	$B \rightarrow D$
UKQCD(94)	0.9($\frac{1.0}{5}$)	1.2($\frac{7}{3}$)	h_{A_1} ($B \rightarrow D^*$)
LANL(96)	0.97(6)		$D \rightarrow D$
H-M(95)		0.70(17)	HQET
UKQCD(99)	1.11($\frac{30}{16}$)	1.16($\frac{23}{13}$)	$B \rightarrow D$
GLOK(99)		1.5(5)	NRQCD, $B \rightarrow B$
experiments			
ALEPH	0.31(17)(8)		
DELPHI	0.78(19)(4)		
OPAL	0.42(17)(5)		
ARGUS	1.17(24)(6)		
CLEO	0.84(12)(8)		

$$3. \quad B \rightarrow \pi \ell \nu, \rho \ell \nu \quad \text{rare decays} \Rightarrow |V_{ub}|$$

Lattice calculation is difficult:
 momenta of $B, \pi (\rho) \ll 1/a \rightarrow$ large q^2 only
 \Rightarrow useful if differential decay rate can be measured

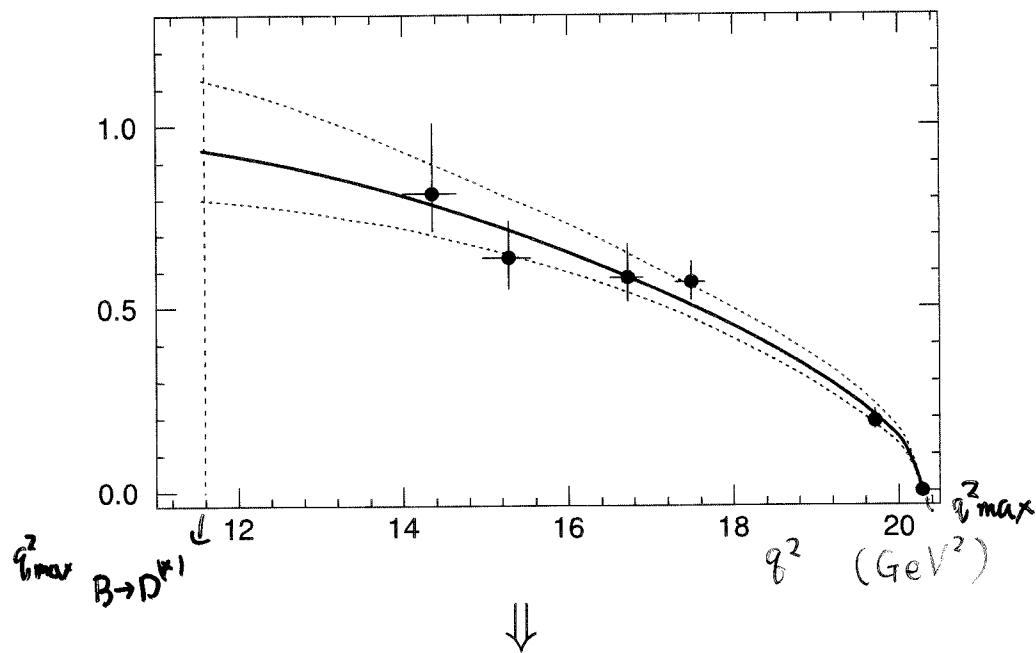
$$(1) \quad B \rightarrow \rho \ell \nu \quad (\text{related to } B \rightarrow K^* \gamma \text{ by HQS})$$

differential decay rate

$$\frac{d\Gamma}{dq^2}(B \rightarrow \rho \ell \nu) \frac{10^{12}}{|V_{ub}|^2} \propto c^2(1 + b(q^2 - q_{\max}^2)) \cdot (\text{phase space})$$

Quenched approximation, $a \neq 0$

UKQCD96



$$c = 4.6 \pm 7 \text{ GeV} \quad b = (-8_{-6}^{+4}) \times 10^{-2} \text{ GeV}^{-2}$$

(2) $B \rightarrow \pi \ell \nu$

$$\langle \pi(k) | \bar{u} \gamma_\mu b | B(p) \rangle = f^+(q^2) \left[(p+k)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \right] + f^0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu$$

differential decay rate

$$\frac{d\Gamma}{dq^2}(B \rightarrow \pi \ell \nu) \propto |V_{ub}|^2 |f^+(q^2)|^2$$

$f^0(q^2)$ is negligible in the decay rate ($q_\mu L_\mu \rightarrow m_l$)

Status of $f^+(q^2)$

- consistent with the pole dominance model around $q^2 = q_{\max}^2$
- chiral extrapolation ($m_\pi \rightarrow 0$) is subtle:



- pre-mature for a detailed comparison with experimental data
- need more works !

3. Impact on determination of CKM matrix

Unitarity Triangle

1. $b \rightarrow u$ transition

$$\sqrt{\bar{\rho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|$$

2. ϵ (K_0 - \bar{K}_0 mixing)

$$\bar{\eta} [(1 - \bar{\rho}) A^2 \eta_2 S_0(x_t) + P_0(\epsilon)] A^2 \hat{B}_K = \frac{|\epsilon|}{\lambda^{10} C_\epsilon}$$

3. B_d^0 - \bar{B}_d^0 mixing

$$\sqrt{(1 - \bar{\rho})^2 + \eta^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|$$

$$\Delta M_d = \frac{G_F^2}{6\pi^2} \eta_B m_B (\hat{B}_{B_d} F_{B_d}) m_W^2 S_0(x_t) |V_{td}|^2$$

4. B_s^0 - \bar{B}_s^0 mixing

$$\sqrt{(1 - \bar{\rho})^2 + \eta^2} \geq \xi \sqrt{\frac{10.2/ps}{(\Delta M_s)_{min}}}$$

Input parameters from experiments

quantity	value	error	remark
$ V_{cb} $	0.040	0.002	$B \rightarrow D^* \ell \nu$
$ V_{ub} $	$3.56 \cdot 10^{-3}$	$0.56 \cdot 10^{-3}$	$b \rightarrow u$
$ \epsilon $	$2.280 \cdot 10^{-3}$	$0.013 \cdot 10^{-3}$	
ΔM_d	0.471 ps^{-1}	0.016 ps^{-1}	
ΔM_s	$> 12.4 \text{ ps}^{-1}$		
m_t	165 GeV	5 GeV	

Input parameters from theories

quantities	Buras99	Lattice
\hat{B}_K	0.80(15)	0.87(6)
F_B		210(30) MeV
\hat{B}_{B_d}		1.28(0.08)
$\sqrt{\hat{B}_B} F_B$	200(40) MeV	240(23) MeV
ξ	1.14(8)	1.17(6)

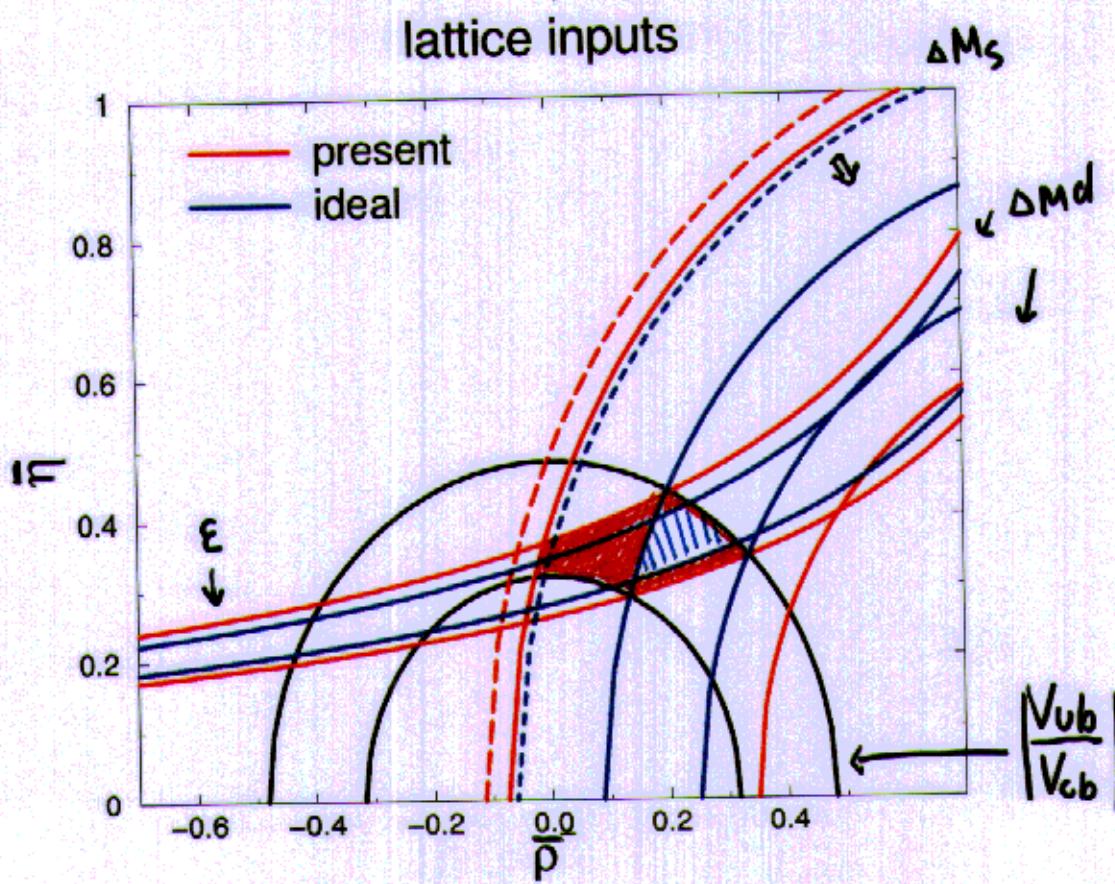
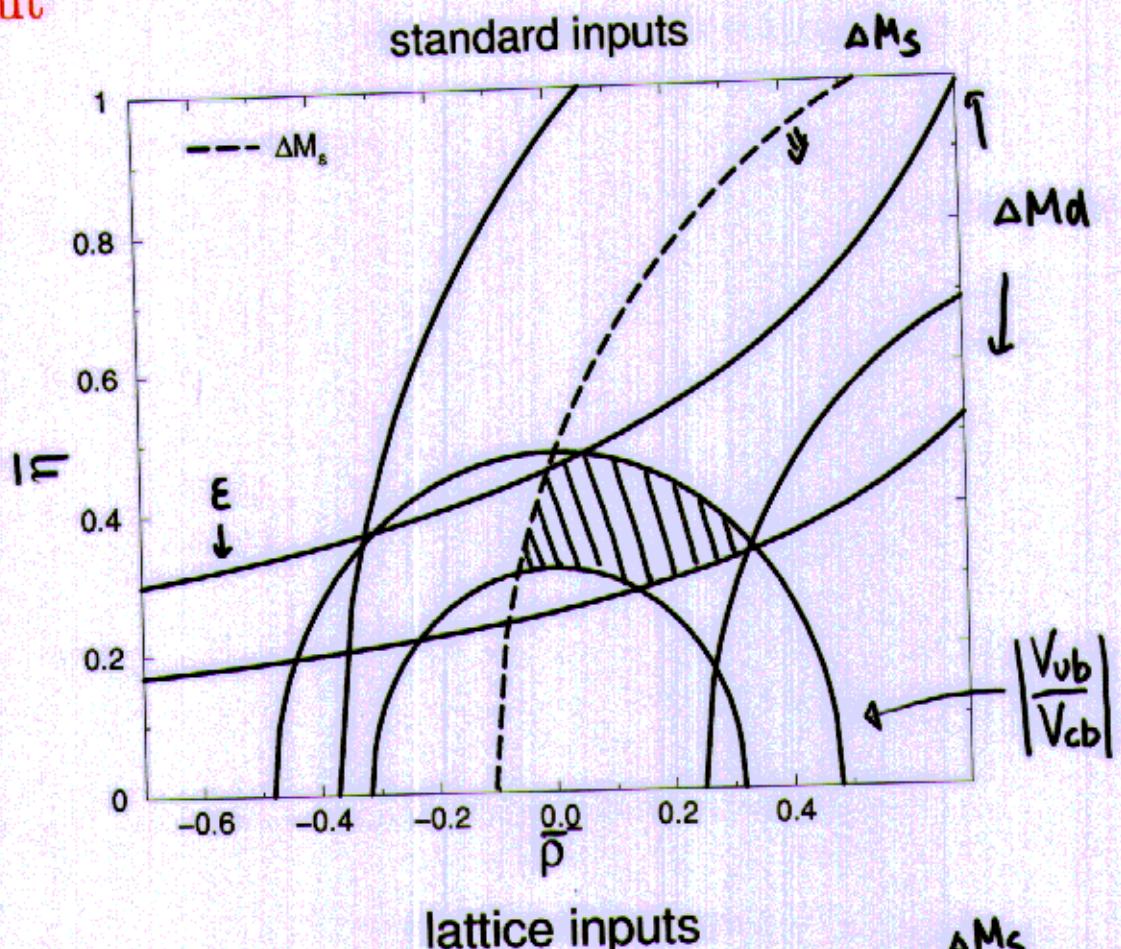
Task of lattice QCD

- reduce errors of $\sqrt{\hat{B}_B} F_B$
- determine central values of B_K , $\sqrt{\hat{B}_B} F_B$ well in continuum limit of full QCD

Task of experiments

- reduce errors of V_{cb} and V_{ub}
- measure ΔM_s

Output



Summary and Outlook

Lattice can contribute to the determination of CKM

- $m_s(m_c) = 100(15)$ ($N_f = 2, a \rightarrow 0$)
smaller than 130(25) MeV often referred
increases the value of ϵ'/ϵ
- $\hat{B}_K = 0.87(6)$ ($a \rightarrow 0$)
full QCD calculations are called for
- $F_B = 210(30)$ GeV ($N_f = 2$)
larger than quenched values
- $B_B(m_b) = 0.80(5)$ (static, $a \neq 0$)
mass-dependence has to be estimated
- Form factors of semileptonic decays
method for $D \rightarrow K^{(*)}, \pi, \rho$ are well-established
promising method for $F_{B \rightarrow D^{(*)}}(1) \rightarrow V_{cb}$
need more work for $B \rightarrow \rho, \pi$

Systematic Continuum limit with full QCD for all
these quantities will be the next target

Lists of some other quantities

quantities	status	N_f	results
$\bar{m}_b(\bar{m}_b)$	2-loop Z	0	4.41(5)(10) GeV
	2-lopp Z	2	4.26(3)(4)(10)
	1-loop Z(NRQCD)	2	4.26(4)(3)(5)
$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s}$	NRQCD	0	0.16(3)(4)
$\frac{\tau(B^-)}{\tau(B^0)}$	static	0	1.03(2)(3)
$\frac{\tau(\Lambda_b)}{\tau(B^0)}$	static	0	0.92(1)(1)
M_{B_c}	NRQCD	0	6.386(9)(98)(15)GeV
	NRQCD	0	6.28(20)
H($b\bar{b}g$)	NRQCD	0	$\Delta m=1.542(8)$ GeV
H($\bar{c}cg$)	NRQCD	0	$\Delta m=1.323(13)$