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Atmospheric Neutrinos and the Oscillations Bonanza

I. Beamline; Osc. Phenomenology

II. Latest Data on Atm. Neutrinos

SuperKamiokande ; Soudan 2 and MACRO

- 1) "In-Detector" Neutrino Interactions
- 2) "Below-Detector" ν_μ Interactions
- 3) $\nu_\mu \rightarrow \nu_\tau$ Allowed Region in $\sin^2 2\theta, \Delta m^2$
- 4) Can $\nu_\mu \rightarrow \nu_{\text{sterile}}$ be dominant? (SK New)

III. Sub-dominant $\nu_e \leftrightarrow \nu_\mu$?

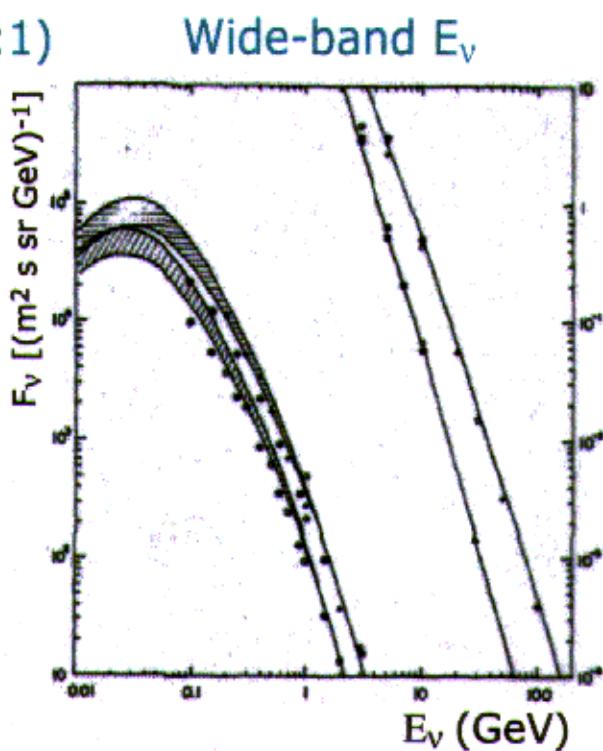
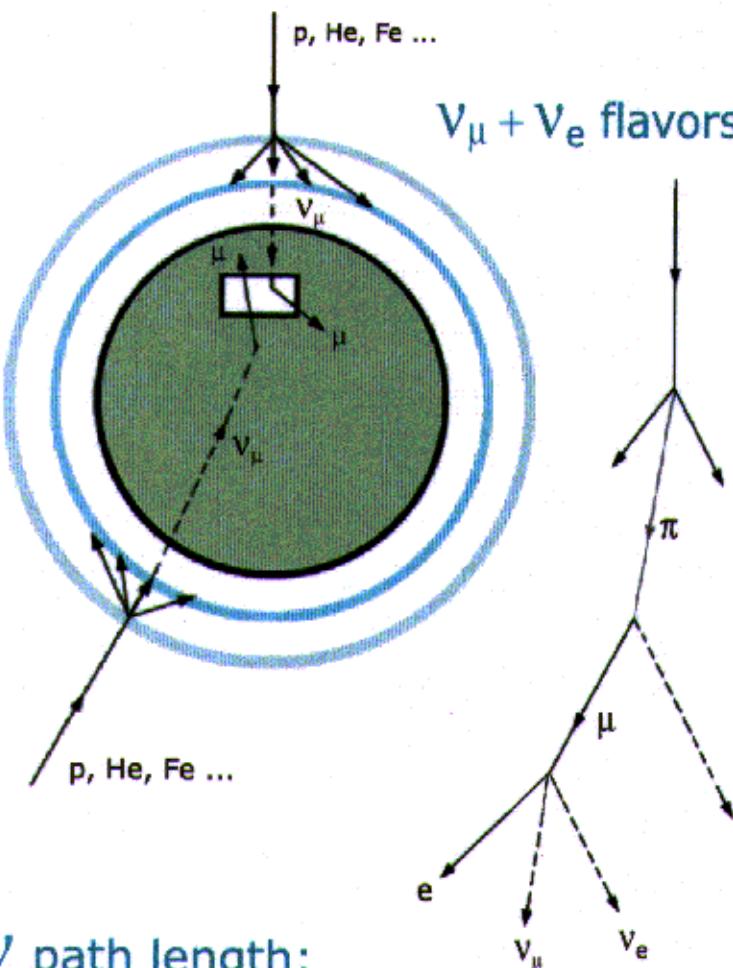
Possibilities with terrestrial matter-induced resonance

IV. Progress with Beam Line Systematics

V. Measurements & Detectors of the Future

Planet Earth - a Splendid Neutrino Beamlne!

$\Phi_\nu(\text{No osc.}) \simeq \text{up/down symmetric}$

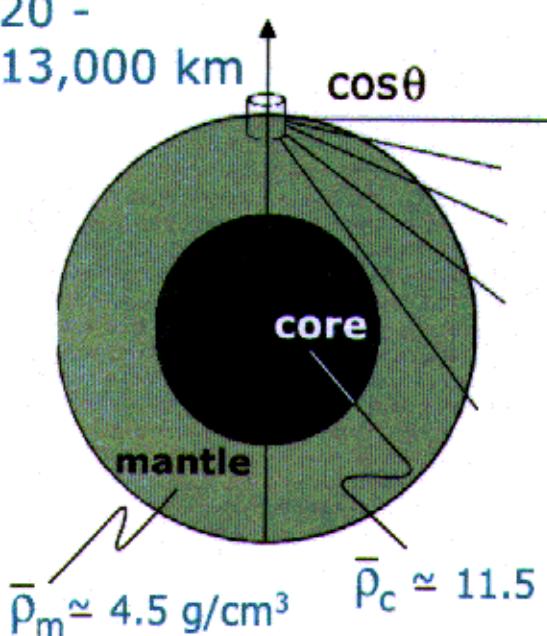


ν path length:

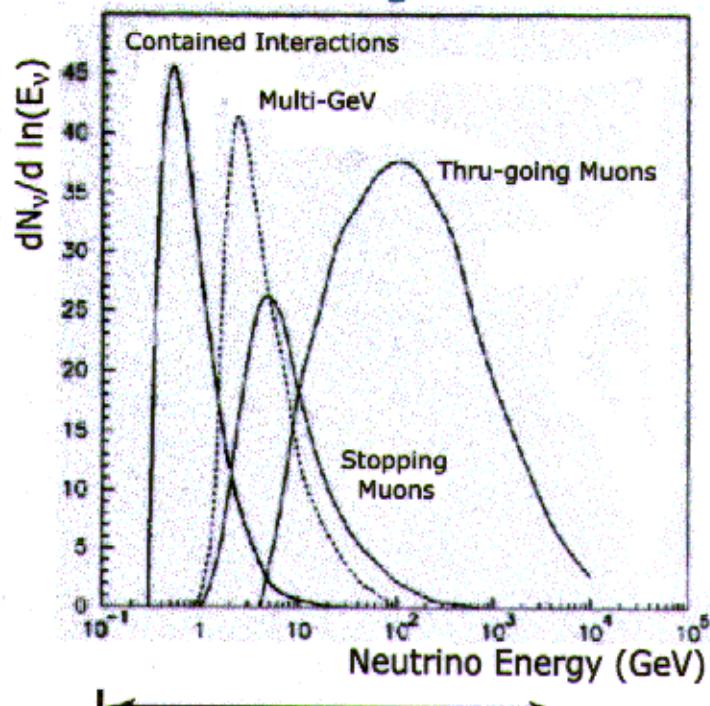
20 -

13,000 km

$\cos\theta$



Counting rates:



Neutrino Oscillations:

For three-flavor mixing among active neutrinos, the weak eigenstates (ν_e , ν_μ , ν_τ) are related to the mass eigenstates according to

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}.$$

Then the probability of oscillation is

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i=1}^3 \sum_{j=i+1}^3 U_{\alpha i} U_{\beta j} U_{\alpha i}^* U_{\beta j}^* \sin^2 \left[\frac{1.27 \Delta m_{ij}^2 L}{E_\nu} \right].$$

In interesting cases the oscillations decouple so that they are approximated by a two-neutrino oscillation:

$$\begin{bmatrix} \nu_\alpha \\ \nu_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix},$$

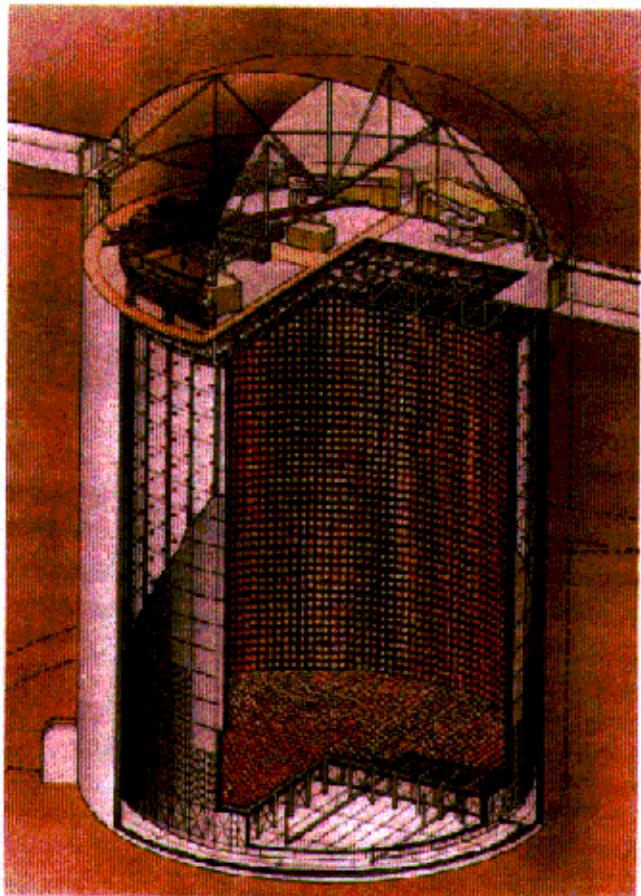
and

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta) \sin^2 \left[\frac{1.27 \Delta m_{12}^2 L}{E_\nu} \right]$$

with

$$\Delta m^2 = |m_1^2 - m_2^2| \text{ in eV}^2, L \text{ in km, } E_\nu \text{ in GeV.}$$

SuperKamiokande



50 kton H₂O Cherenkov detector

Two concentric cylinders -

INNER DETECTOR:

22.5 kton fiducial volume
11,146 20-inch PMTs
inward-looking

OUTER DETECTOR:

1,885 8-inch PMTs
outward-looking
- veto entering events,
tag exiting tracks

- ν flavor: μ tracks yield "sharp" rings
- electron tracks yield "fuzzy" rings
- ⇒ CC(ν_μ), CC(ν_e) discrimination

Event vertex and particle direction are determined using PMT timing.

Momentum is determined from observed number of photoelectrons.

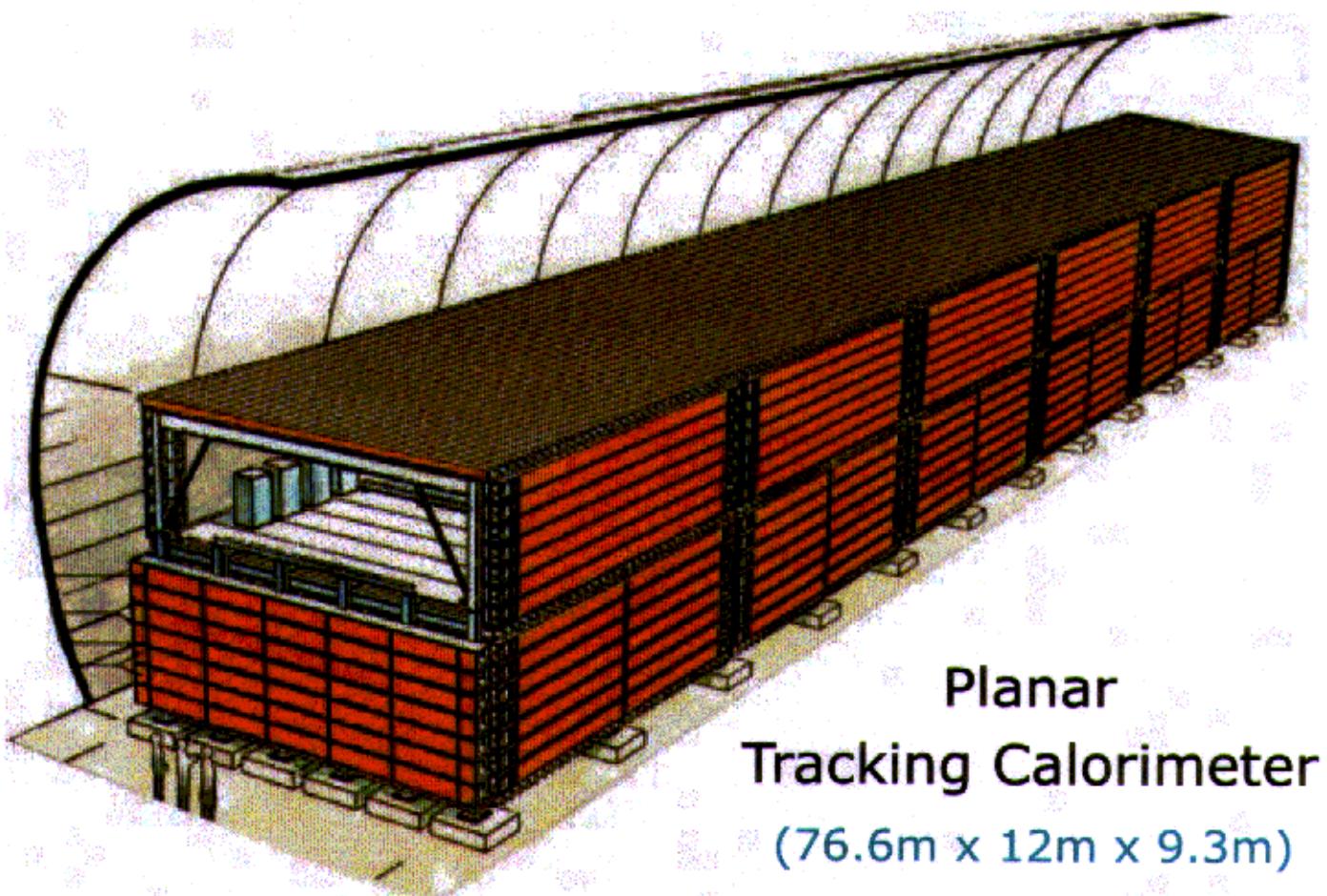
Analyzed exposure:

FC + PC events: 848 days = 52 kton years

Up-thru muons: 923 days

Up-stop muons: 916 days

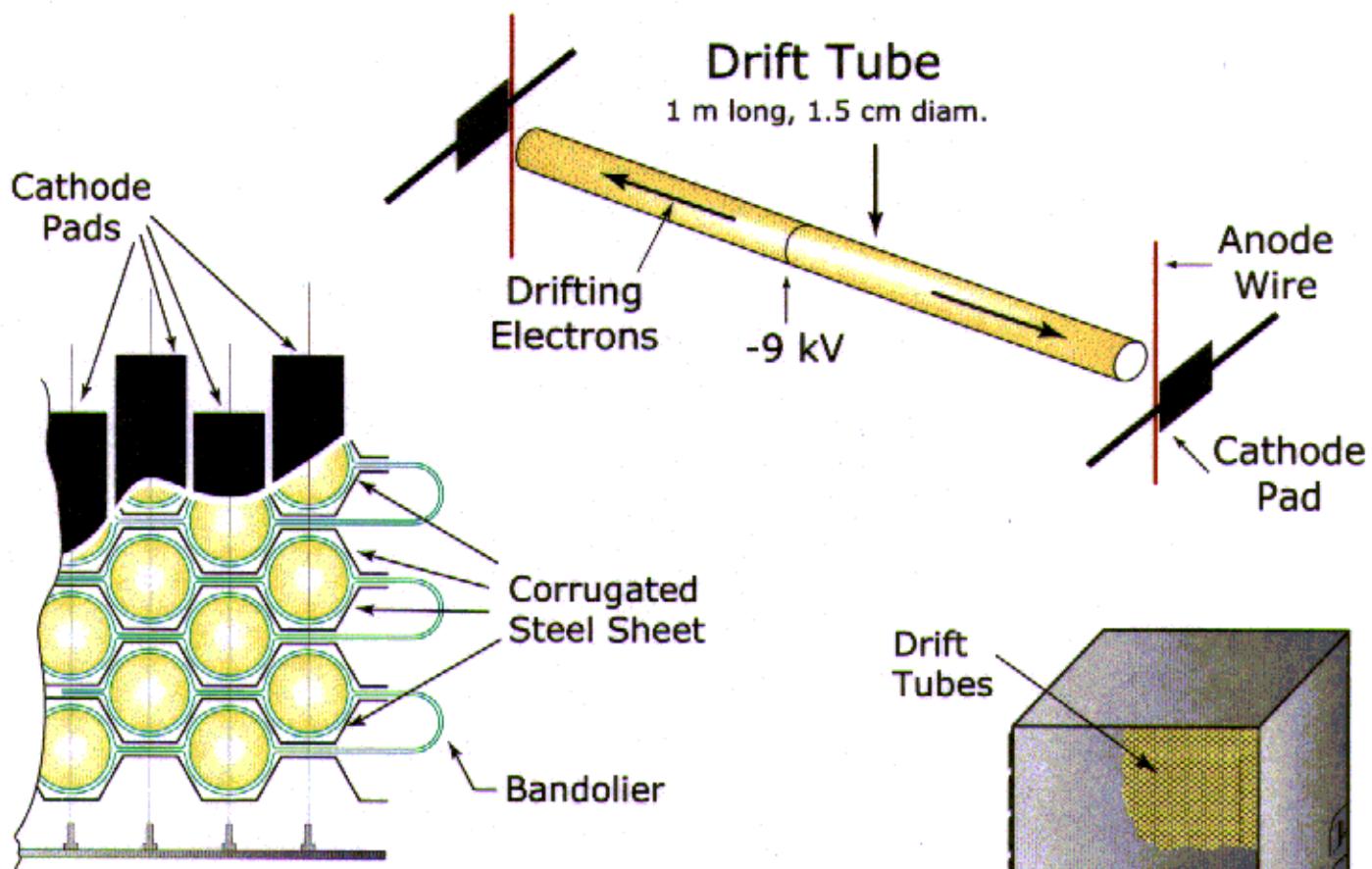
The MACRO Detector



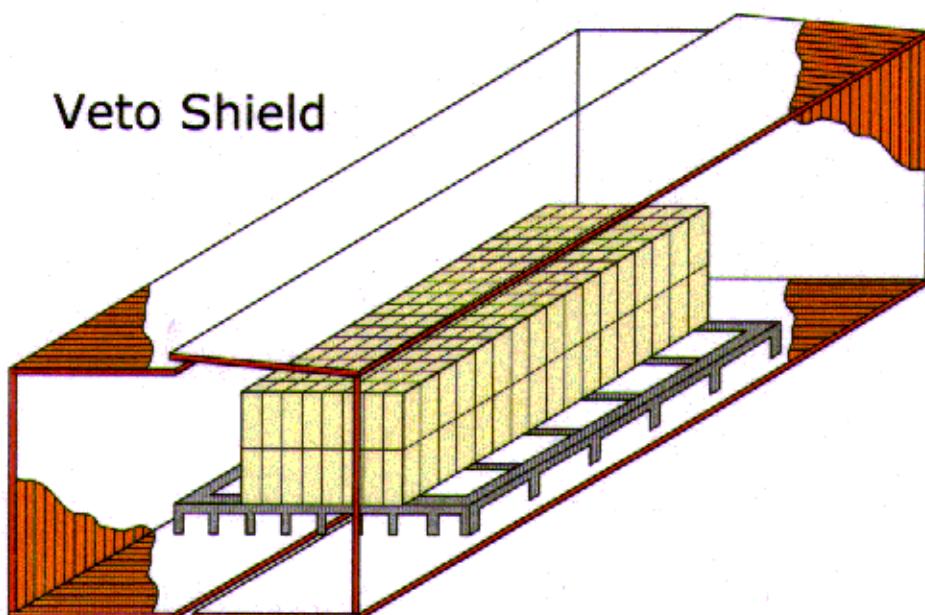
- Tracking:** Alternating layers of crushed rock & streamer tube planes.
Streamer tubes - 3 cm cells, wire & 27° stereo strip readout.
Angular Resolution < 1°.
- Timing:** Liquid scintillation counters
3 horizontal planes & vertical walls
Resolution ~ 0.5 nsec.
- Mass:** ~ 5.3 kilotons
- Overburden:** 3150 hg/cm² - low down-going μ rate.

The Soudan 2 Detector:

Slow-drift time projection chamber



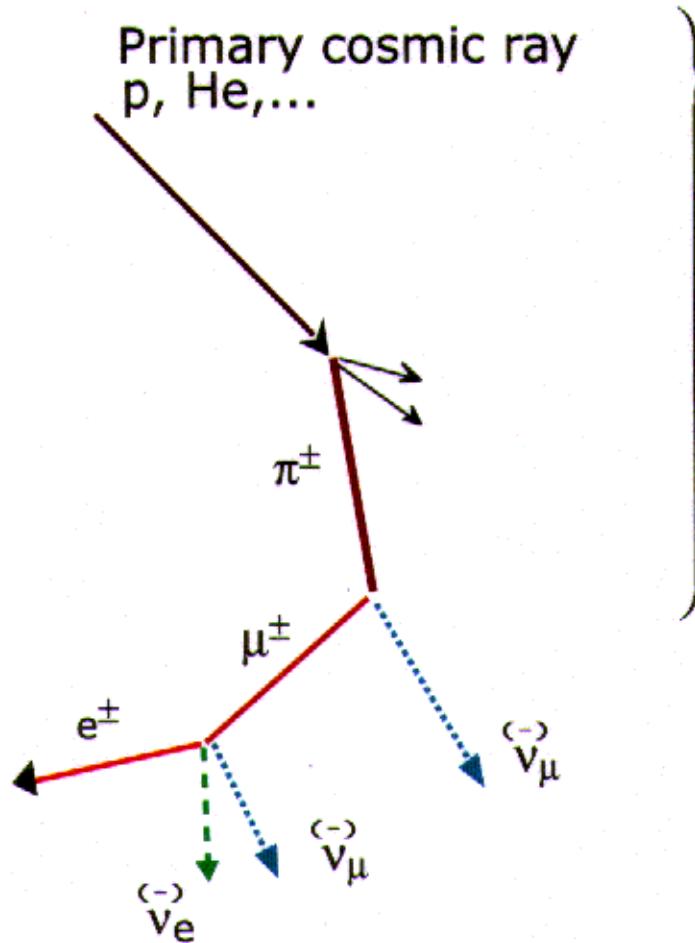
Honeycomb lattice
geometry



4.3 ton Module

Mass: 963 tons
Analyzed exposure: 4.6 fid. kty

Atmospheric Neutrino Flavor Ratio



Uncertainties:

Primary CR flux

Solar cycle, Geomagnetic effects

Particle production in hadronic showers



Absolute ν flux: $\pm 20\%$
However for the flavor ratio: $\pm 5\%$

$$\frac{(\nu_\mu + \bar{\nu}_\mu)}{(\nu_e + \bar{\nu}_e)} \simeq 2$$

Experiments examine the ratio-of-ratios:

$$R = \frac{(\nu_\mu/\nu_e)_{\text{DATA}}}{(\nu_\mu/\nu_e)_{\text{MC (no osc.)}}} \stackrel{?}{=} 1$$

Actually they measure R' :

$$R' = \frac{\left(\frac{\mu\text{-like}}{e\text{-like}} \right)_{\text{Single-ring evts}}}{\left(\frac{\mu\text{-like}}{e\text{-like}} \right)_{\text{MC}}}$$

(SuperK)

or

$$\frac{\left(\frac{\text{tracks}}{\text{showers}} \right)_{\text{Single-prong evts}}}{\left(\frac{\text{tracks}}{\text{showers}} \right)_{\text{MC}}}$$

(Soudan 2)

A Decade-plus of Atm. Flavor Ratios - the Latest: (Preliminary)

SuperKamiokande:

Water Cherenkov

52 kton - yrs.

$$R(\mu/e)_{\text{single-ring}} \begin{cases} = 0.68 \pm 0.02 \pm 0.05 & (\text{Sub-GeV}) \\ = 0.68 \pm 0.04 \pm 0.08 & (\text{Multi-GeV}) \end{cases}$$

↓
Systematics - limited

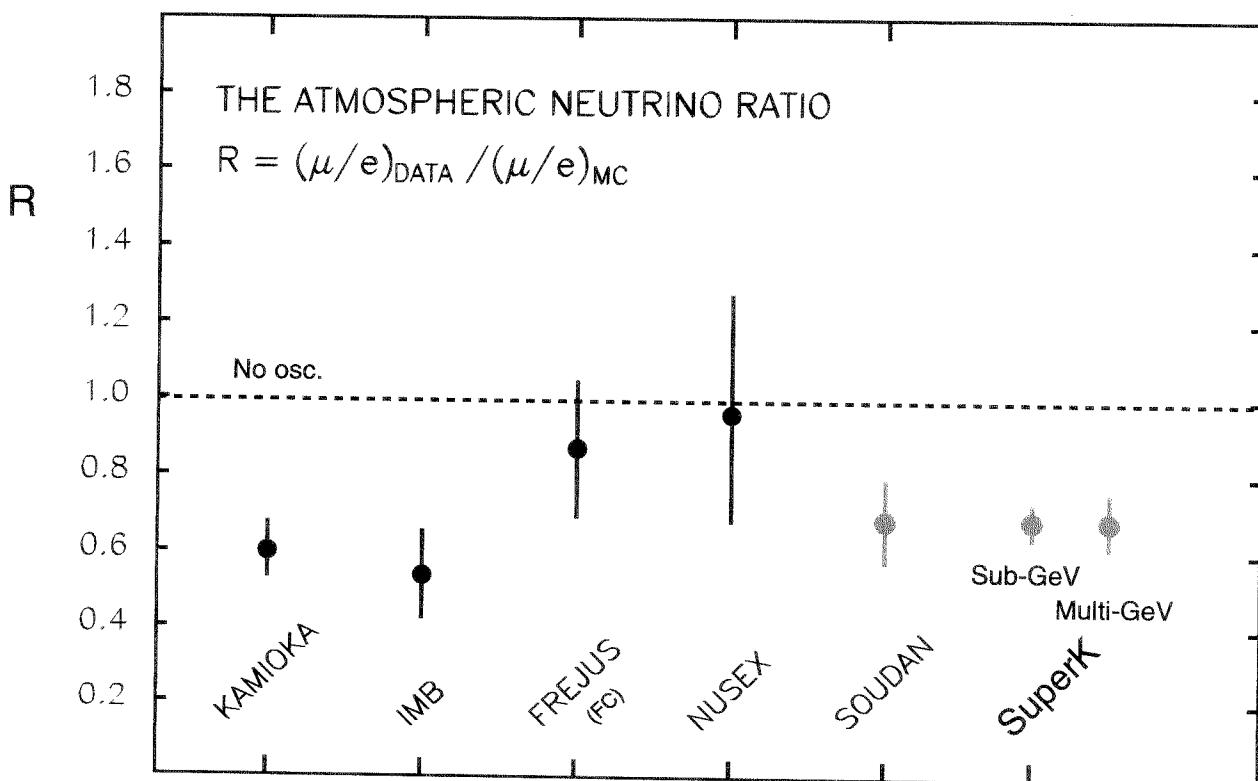
Soudan 2:

Iron Tracking Calorimeter

4.6 kton - yrs.

$$R(\mu/e)_{\text{single-trk, shwr}} = 0.68 \pm 0.11 \pm 0.06 \quad (\text{Mostly Sub-GeV})$$

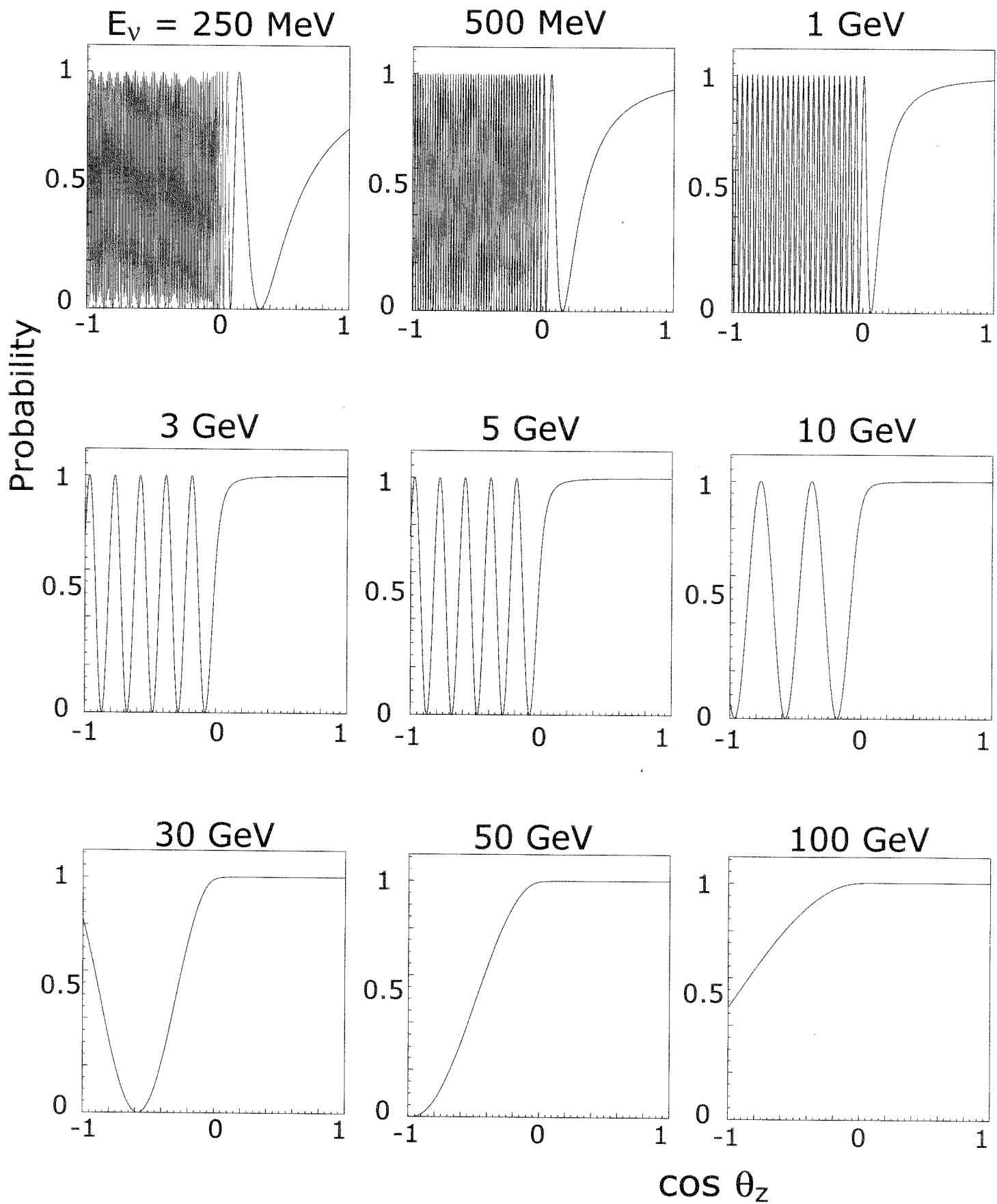
↓
Statistics - limited



Survival Probability for V_{μ} :

$$\sin^2 2\theta = 1.0$$

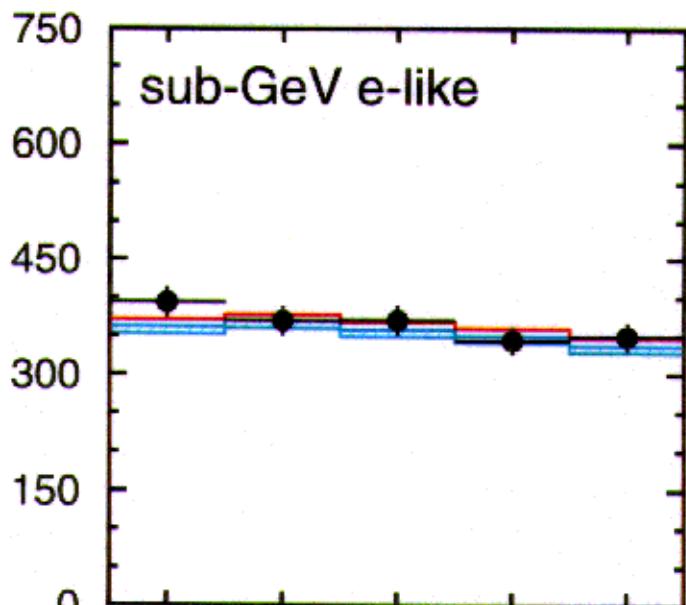
$$\Delta m^2 = 5 \times 10^{-3} \text{ [eV}^2]$$



848 days = 52 kty

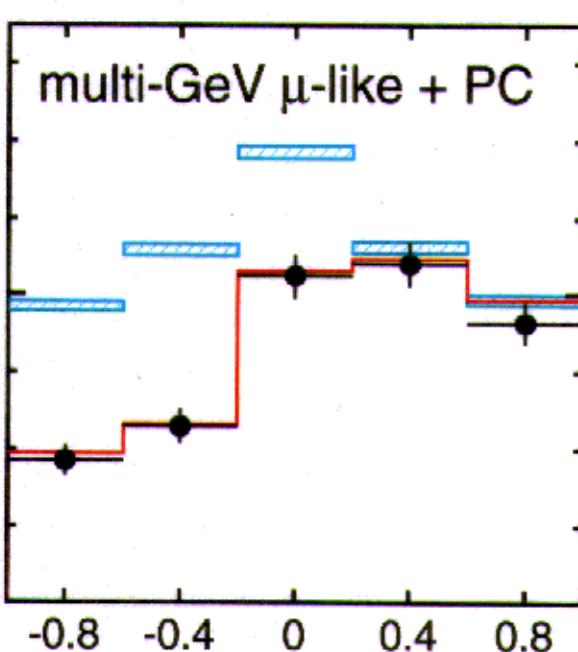
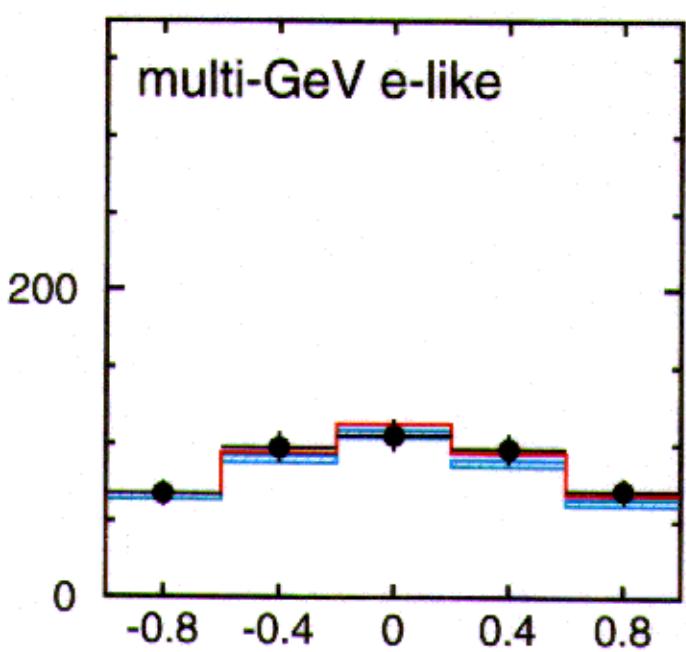
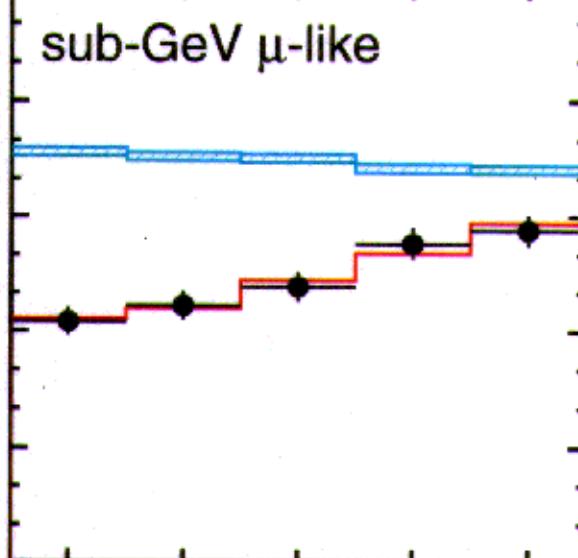
Zenith Angle Distributions:

ν_e 's - No angular distortion



ν_μ 's - "disappear" !

Dependence on $\Theta_z \Rightarrow L$ and E_ν



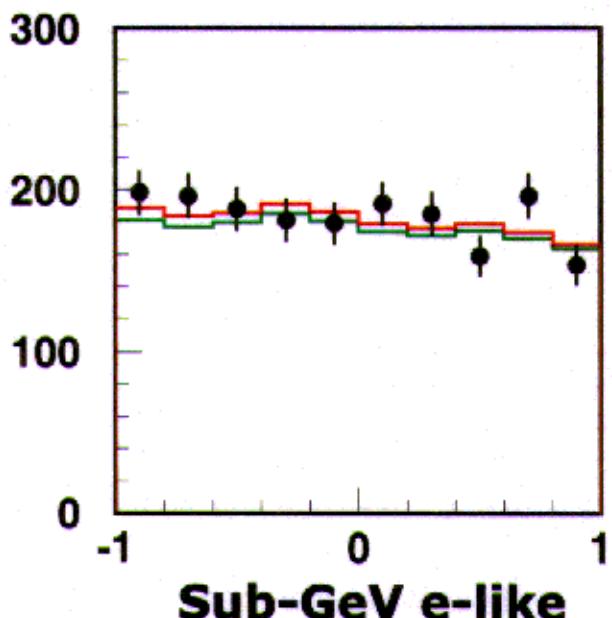
Up-going
↑

$\cos\Theta$

↓ Down-going

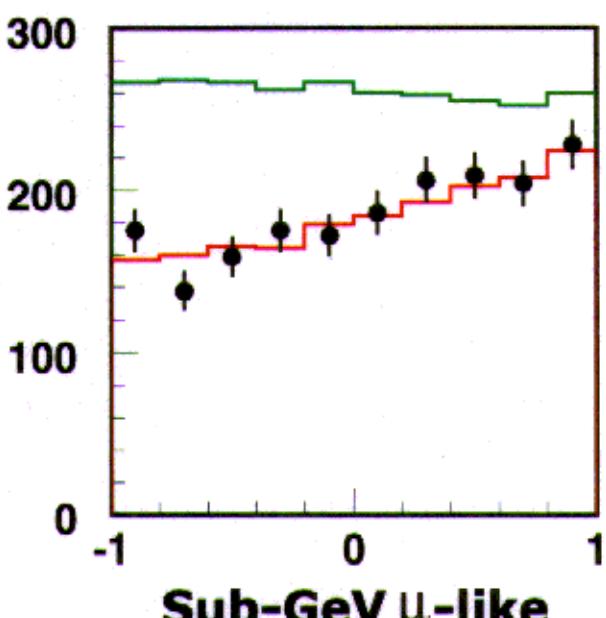
$\nu_\mu \leftrightarrow \nu_\tau$ Best Fit

Zenith Angle - 10 bins:

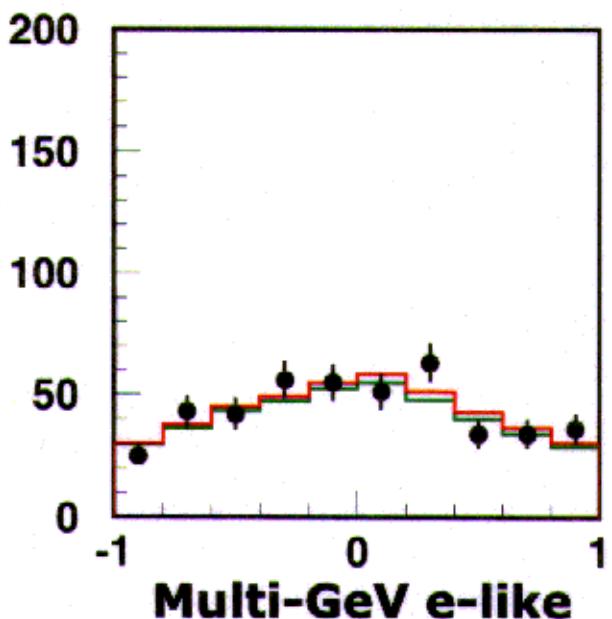


Sub-GeV e-like

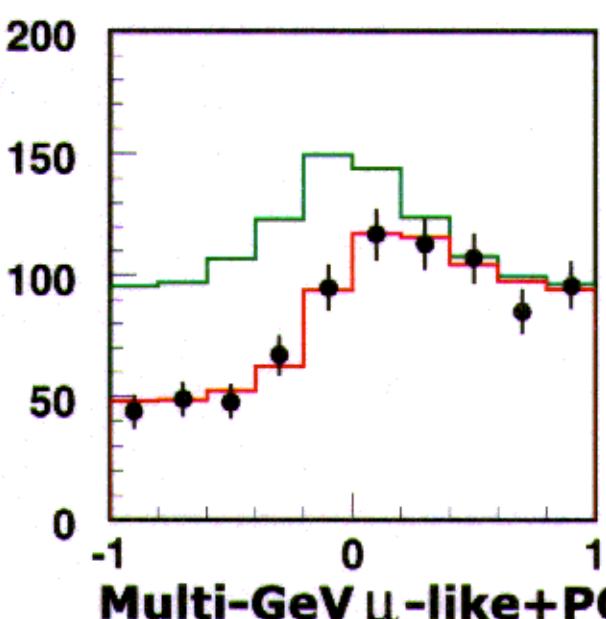
V_μ depletion at all angles including above-horizon



Sub-GeV μ -like



Multi-GeV e-like



Multi-GeV μ -like+PC

V_e - distributions "well-behaved"
No overt hints for
sub-dominant $V_\mu \leftrightarrow V_e$.

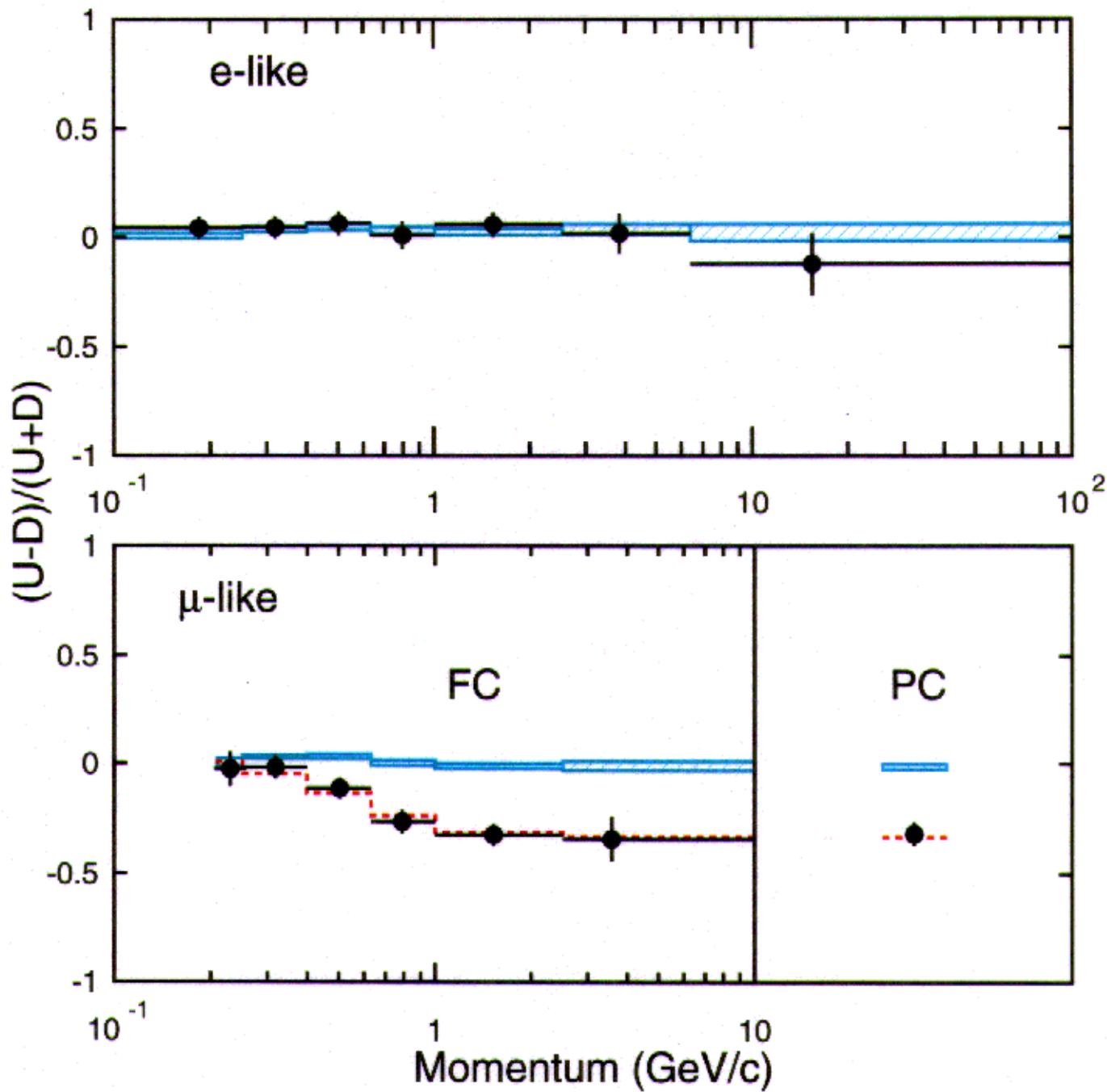
With increasing E_ν ,
 V_μ -depletion moves to
(mostly) below-horizon.

Zenith Angle Asymmetry:

$$A = (U - D)/(U + D)$$

U: $\cos \theta > +0.2$

D: $\cos \theta < -0.2$



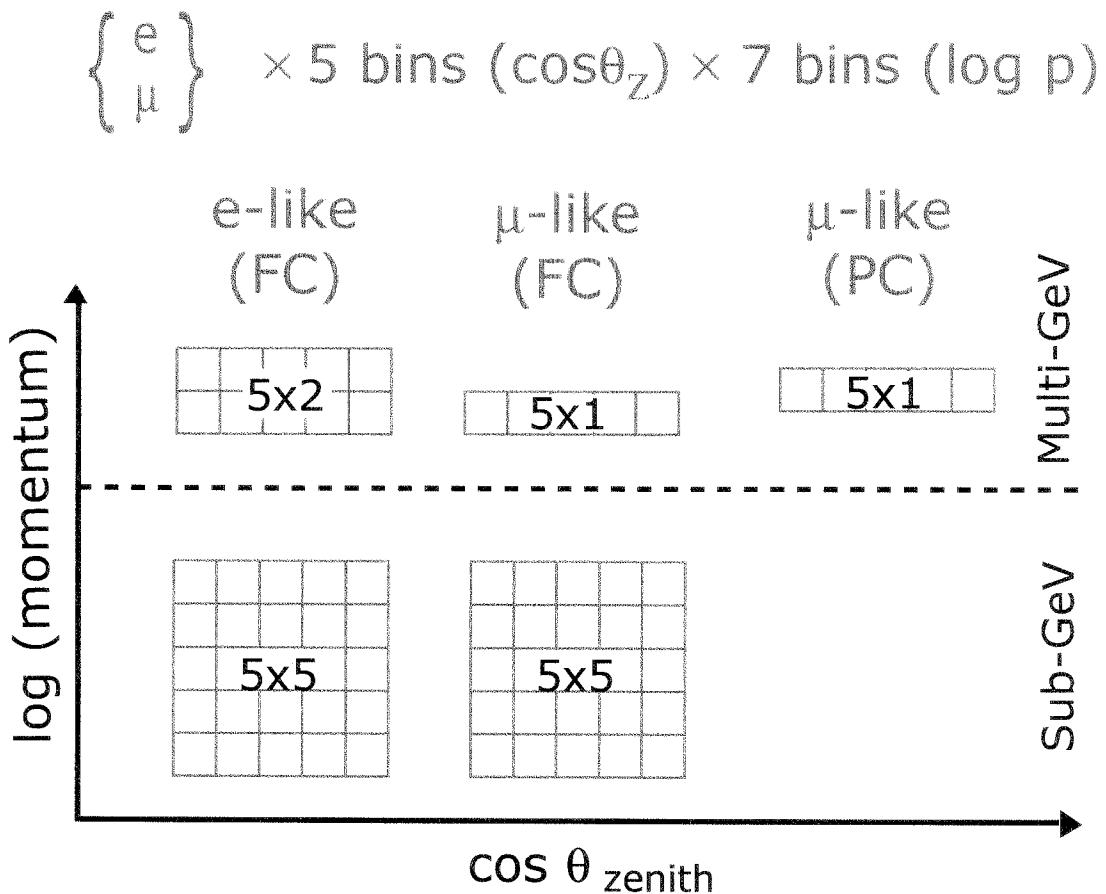
Multi-GeV μ : $A = -0.32 \pm 0.04 \pm 0.01$

↗ asymmetry approaching 8σ !

SuperKamiokande:

χ^2 for Oscillation Parameters Determination

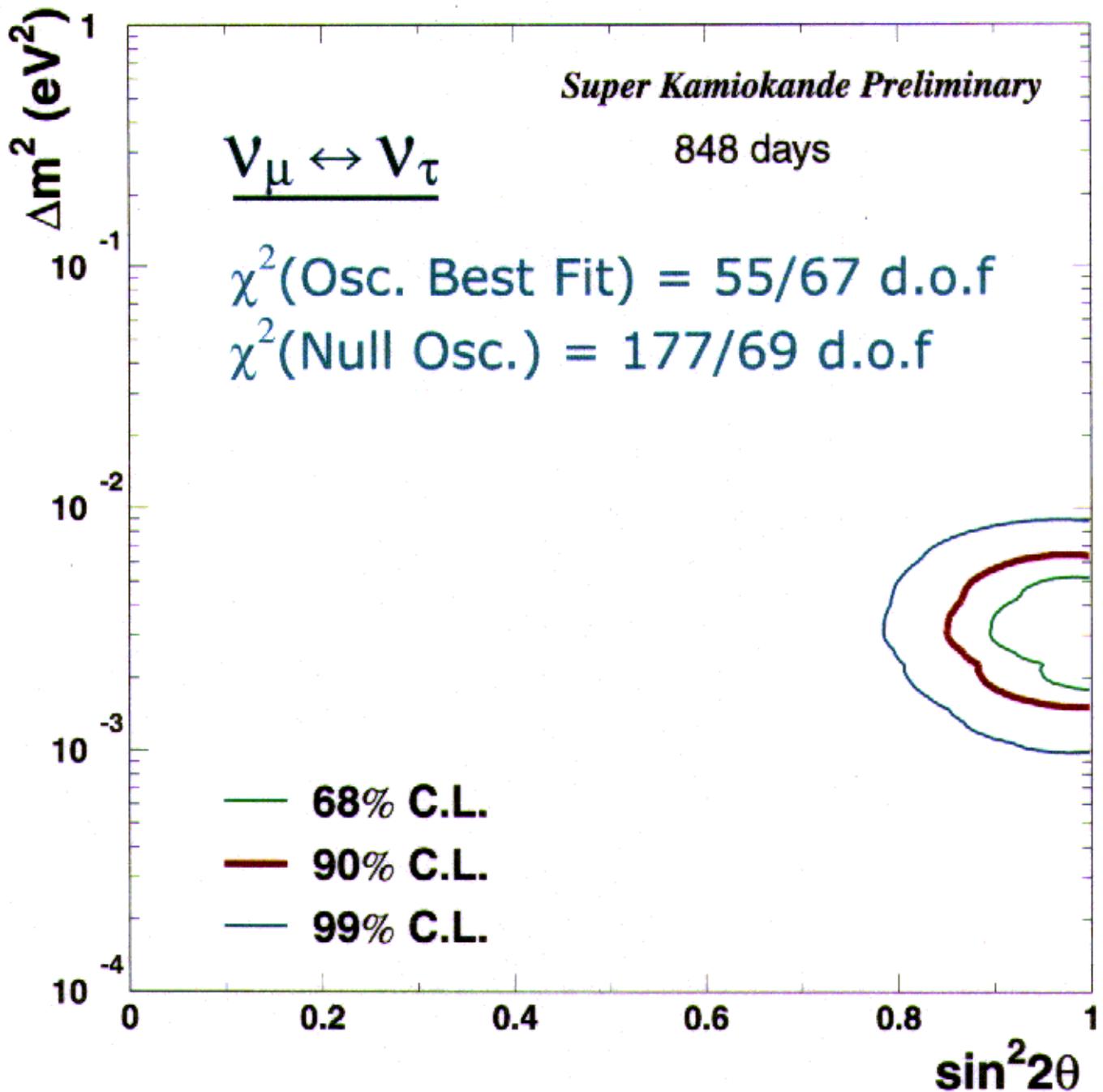
Data is binned:



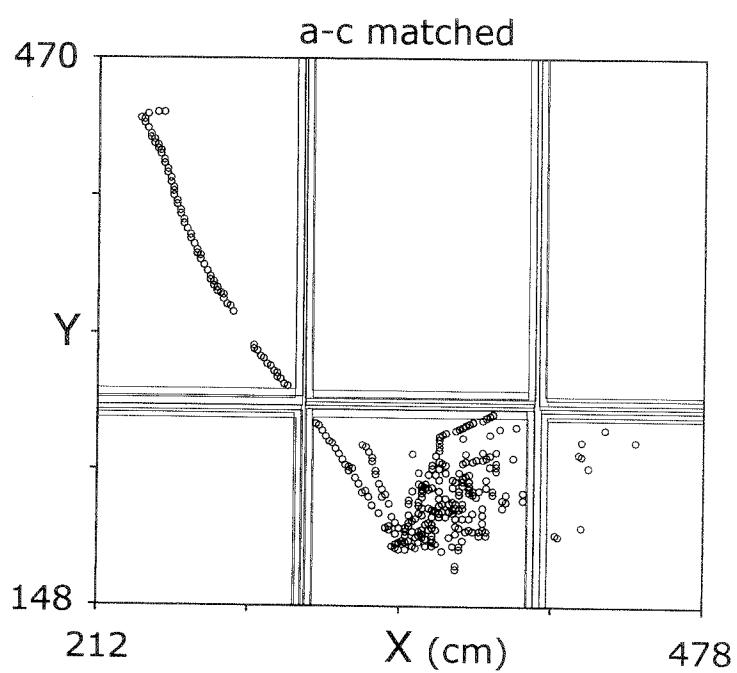
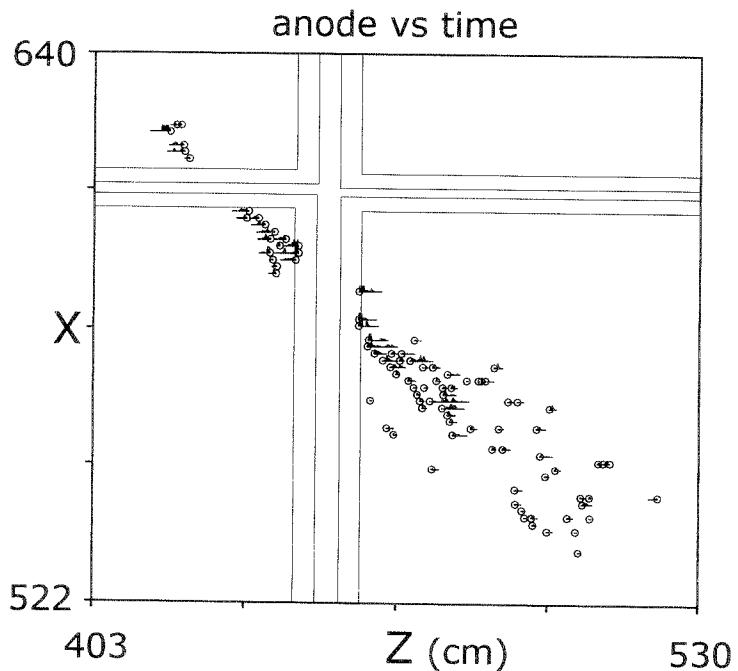
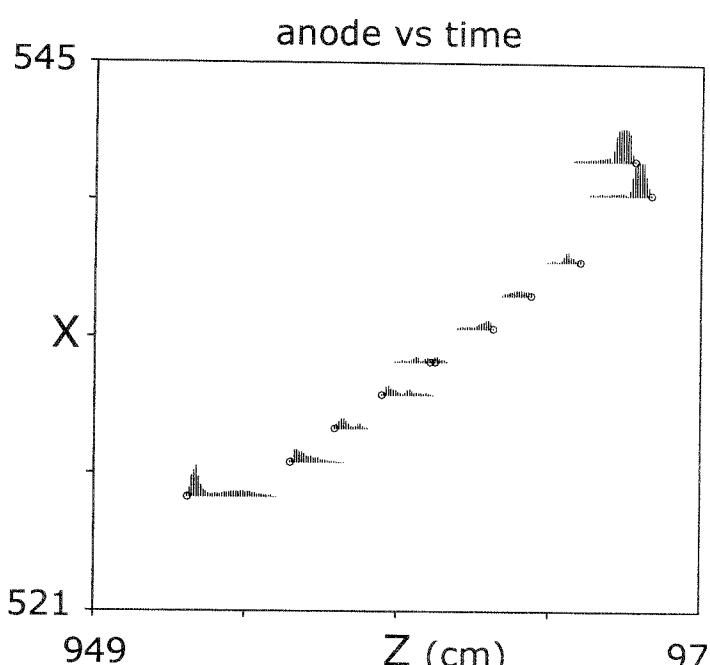
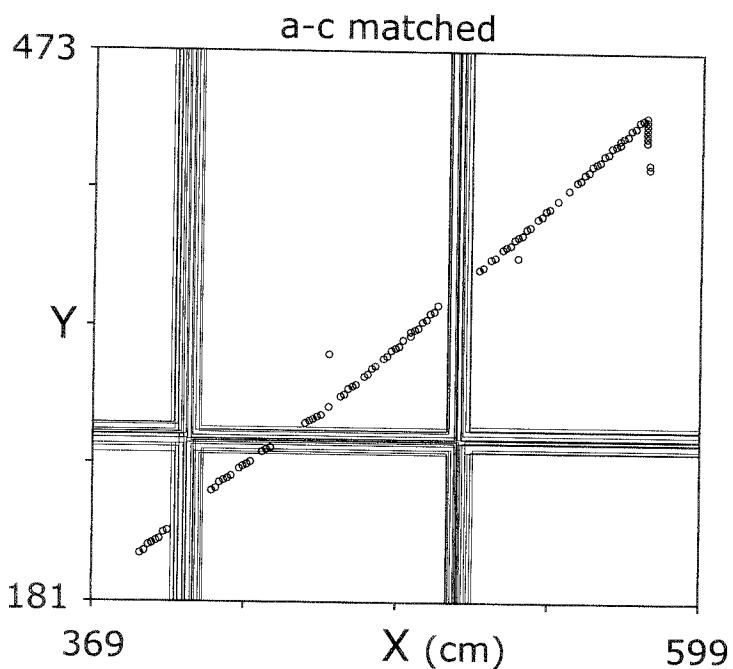
$$\chi^2(\sin^2 2\theta, \Delta m^2, \vec{\epsilon}) = \sum_{i=1,70} \frac{(N_{\text{data}}^i - N_{\text{MC}}^i)^2}{\sigma_i^2} + \sum_j \frac{\epsilon_j^2}{\sigma_j^2}$$

ϵ_j : Estimated systematics, including
 α = overall ν -flux normalization.

Allowed Region: FC & PC Events



Track, Shower, Multiprong Events in Soudan 2:



The High Resolution Sample

1) Quasi-elastics (Tracks, Showers)

$P_{\text{lept}} > 150 \text{ MeV/c}$ if a recoil is measured

Or

$E_{\text{vis}} > 600 \text{ MeV/c}$ if a recoil is absent

2) Multiprong

$E_{\text{vis}} > 700 \text{ MeV}$

$|\sum \vec{p}_{\text{vis}}| > 450 \text{ MeV/c}$ (improve directionality)

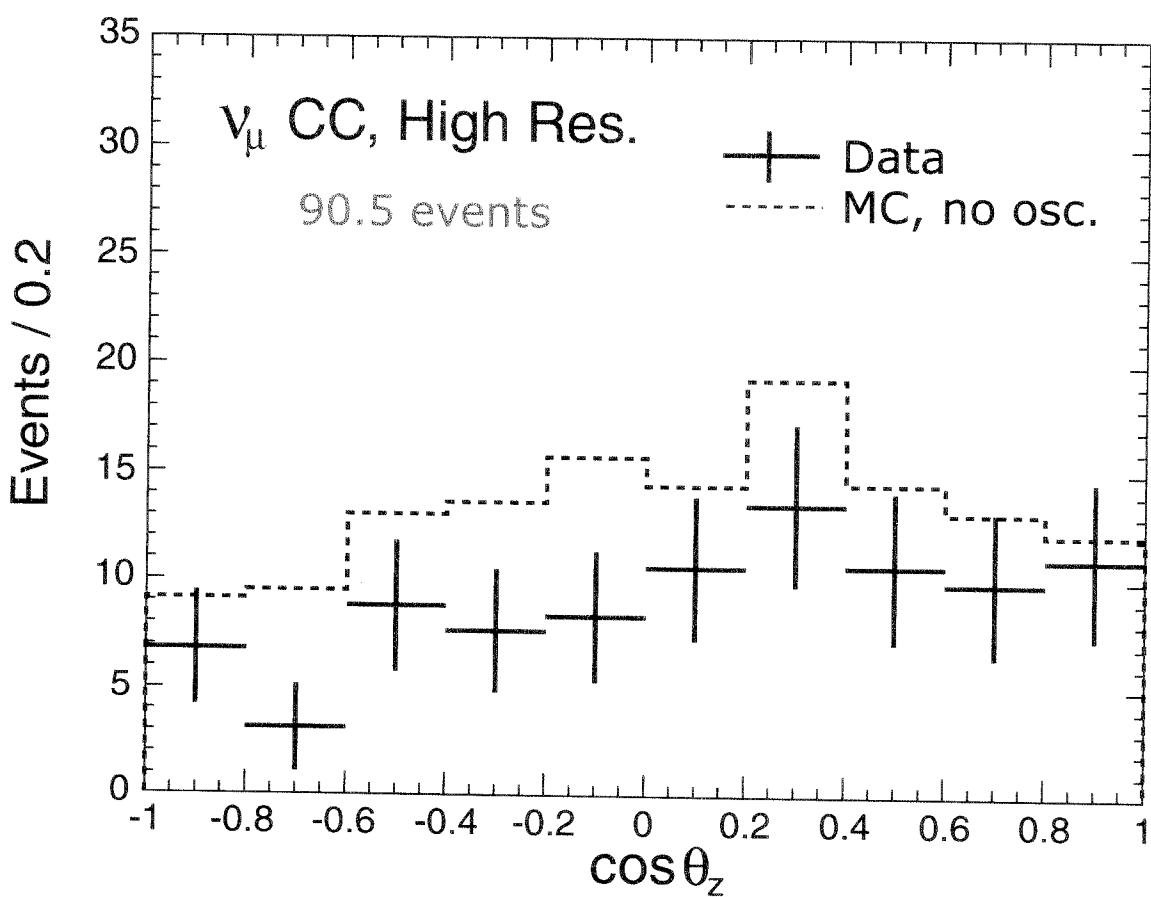
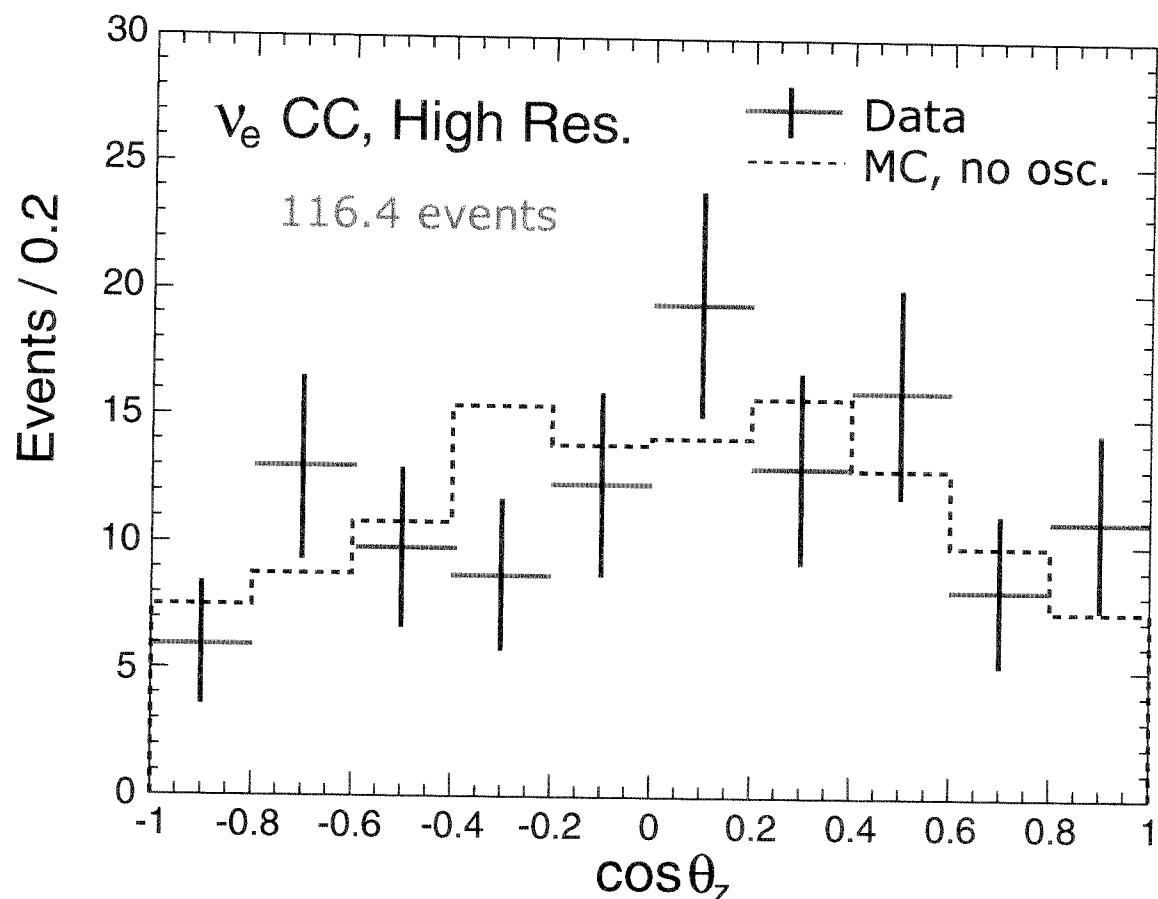
$P_{\text{lept}} > 250 \text{ MeV/c}$ (improve flavor tag)

Resolutions

	$\nu_\mu \text{ CC}$	$\nu_e \text{ CC}$
Energy: $(\Delta E/E)$	20%	23%
Angle: $\angle \vec{p}_\nu(\text{true}) \cdot \vec{p}_\nu(\text{recon})$	33.2°	21.3°
L/E: $ \log(\text{true L/E}) - \log(\text{recon L/E}) $	0.49	0.43

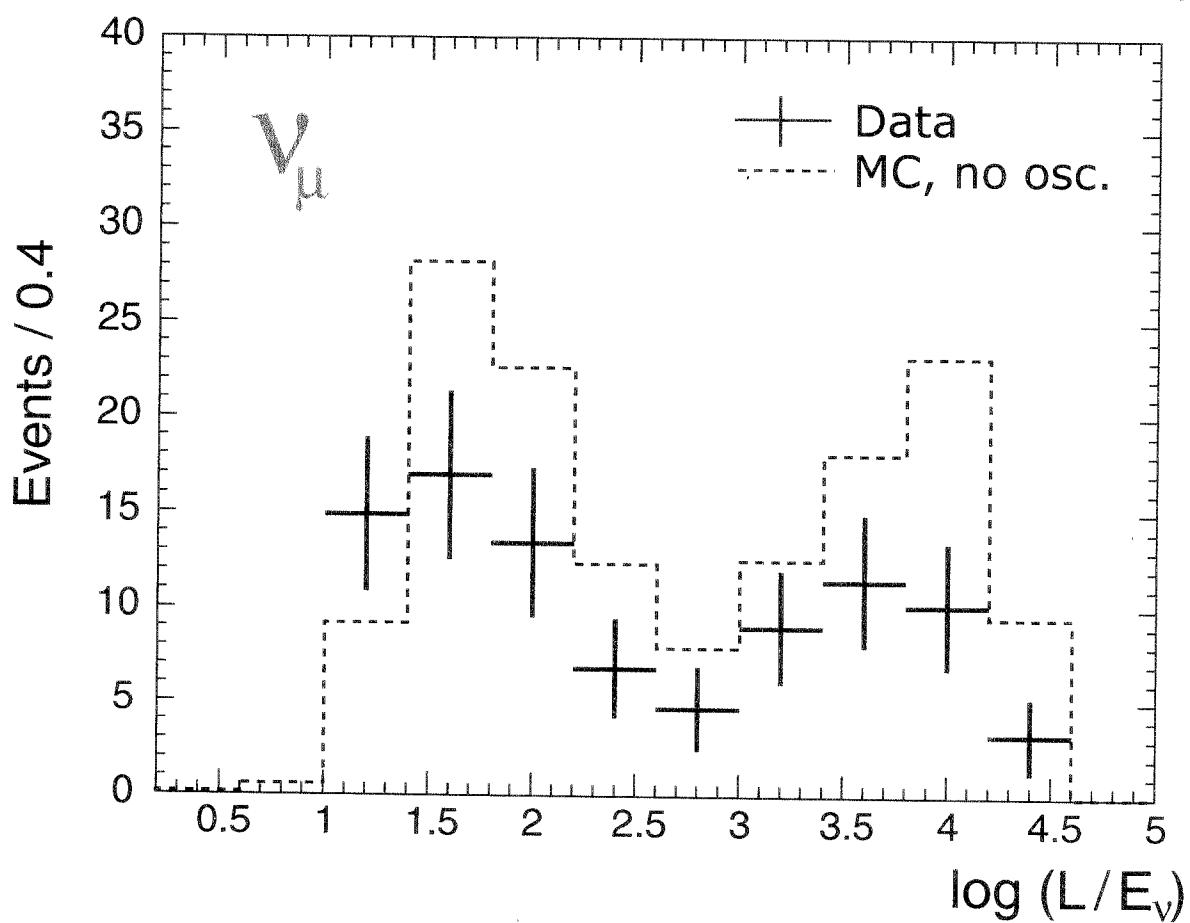
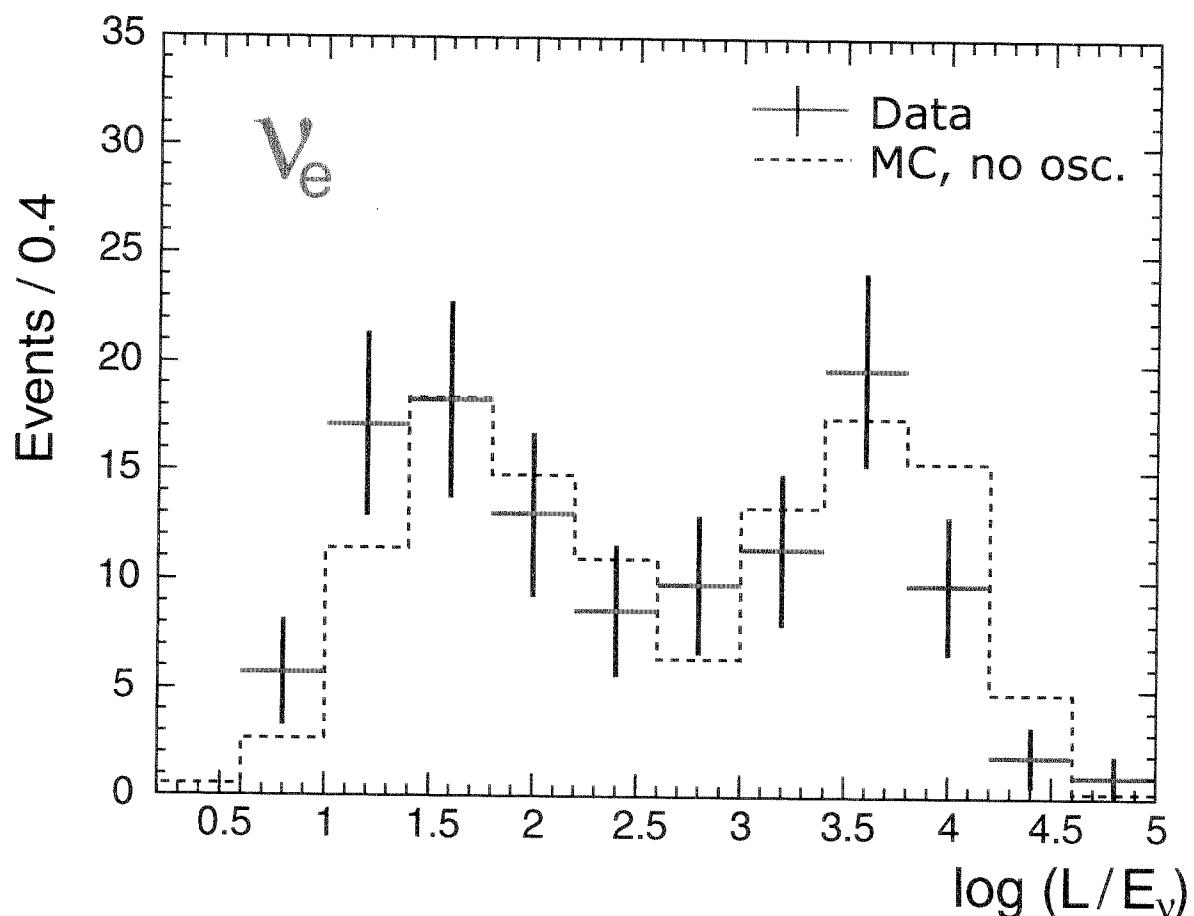
Zenith Angles:

Soudan 2 at 4.6 kty



L/E Distributions:

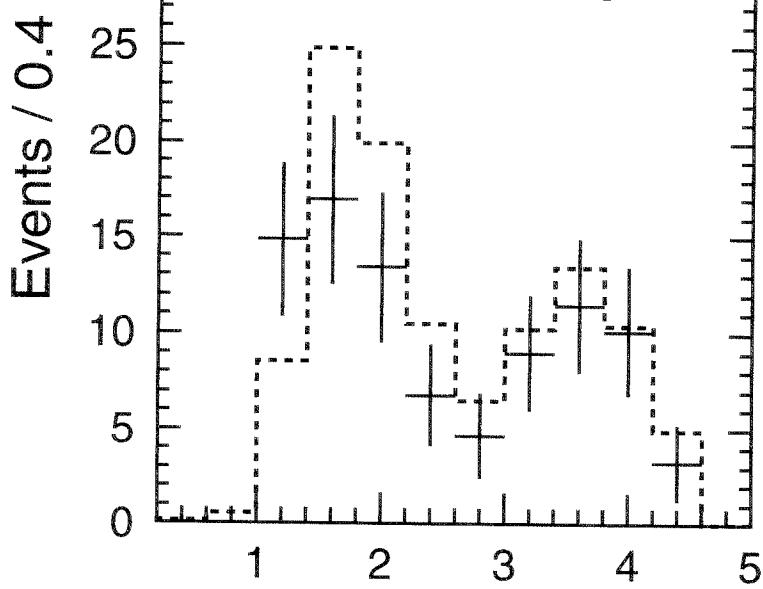
Soudan 2 at 4.6 kty



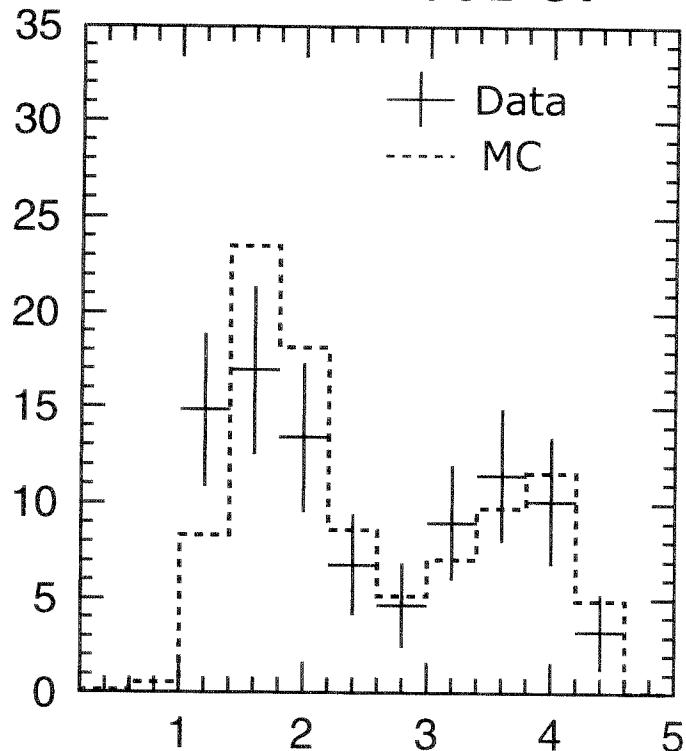
L/E Distributions for ν_μ - Flavor Events:

$$\sin^2 2\theta = 1$$

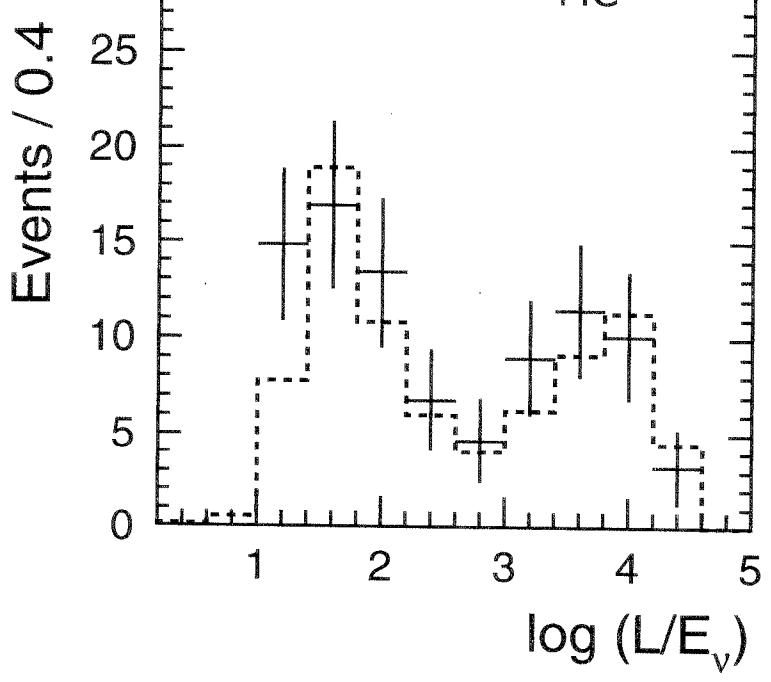
$$\Delta m^2 = 0.0001 \text{ eV}^2$$



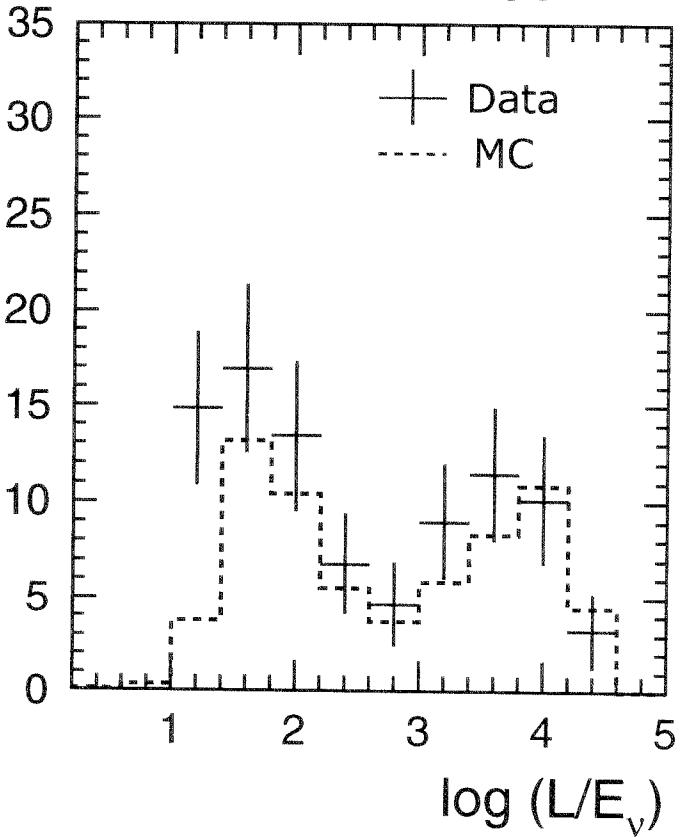
$$\Delta m^2 = 0.001 \text{ eV}^2$$

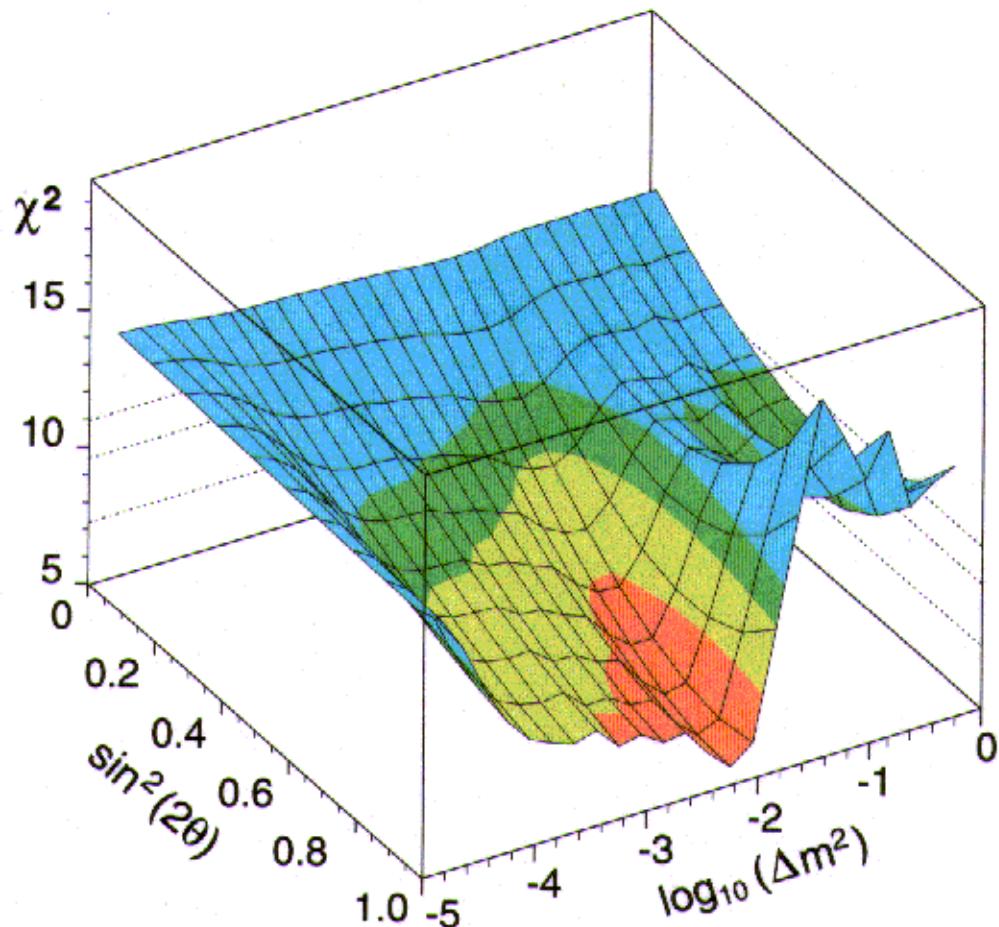


$$\Delta m^2 = 0.008 \text{ eV}^2$$



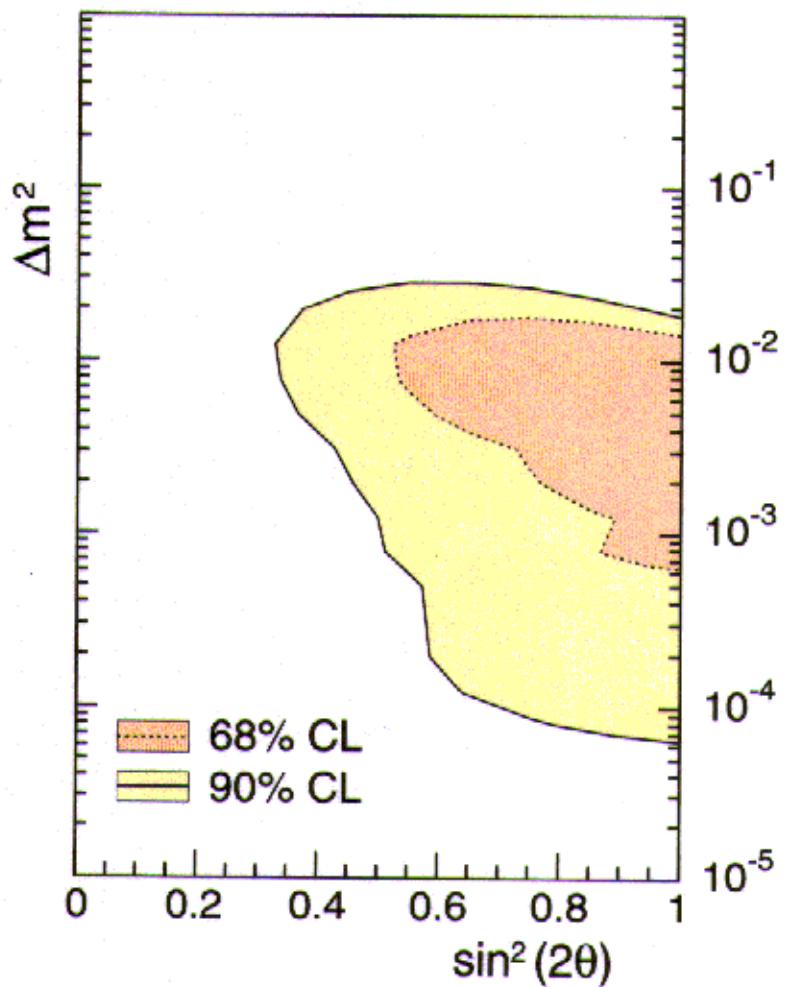
$$\Delta m^2 = 0.1 \text{ eV}^2$$





Contour
boundaries are:
68% CL, 90% CL,
and 95% CL.

χ^2_{\min} at
 $\sin^2 2\theta = 0.95$
 $\Delta m^2 = 0.8 \times 10^{-2} [\text{eV}^2]$
 $f_v = 0.82 * \text{MC (Bartol)}$

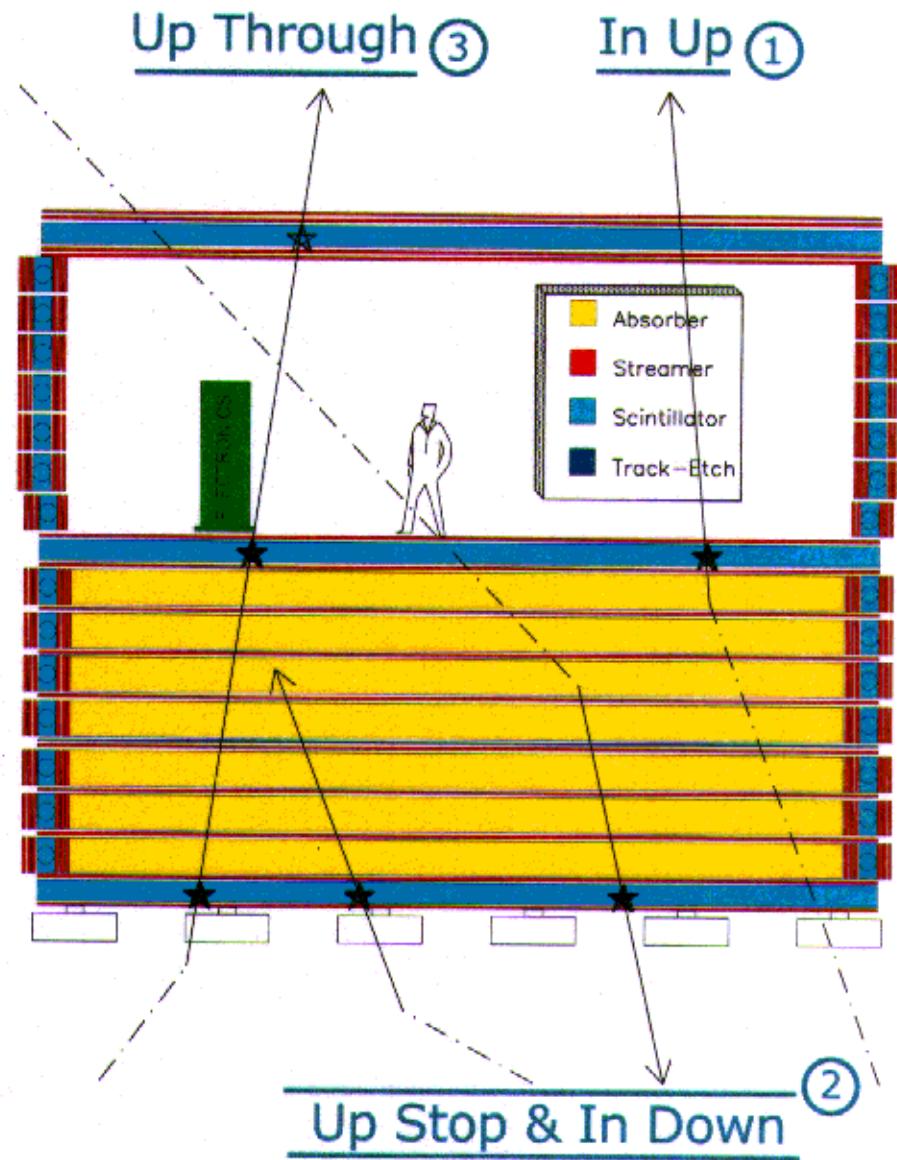


MACRO: ν_μ Event Categories:

PC - Medium Energy:

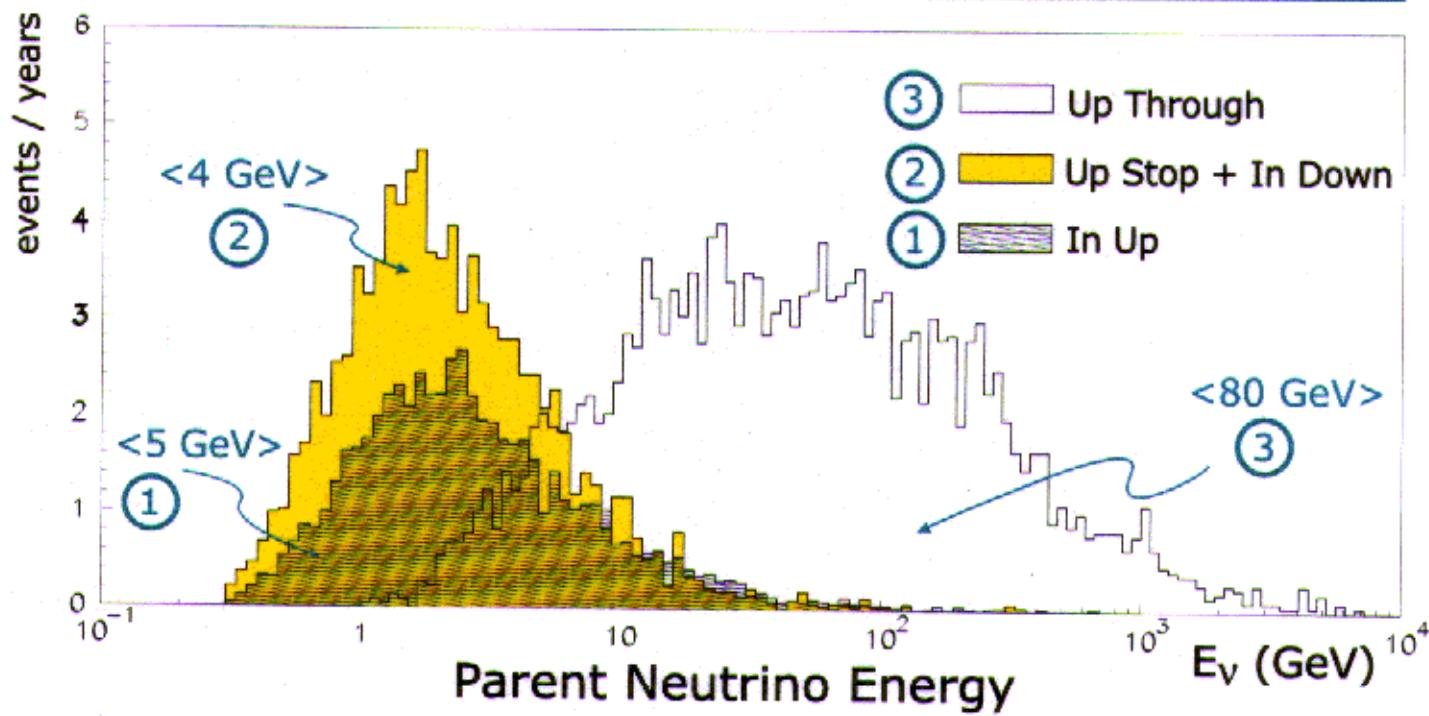
① In-Up Events
(use scintillator T.O.F.)

② Up-Stop & In-Down
(define by topology,
mixture $\sim 50:50$)

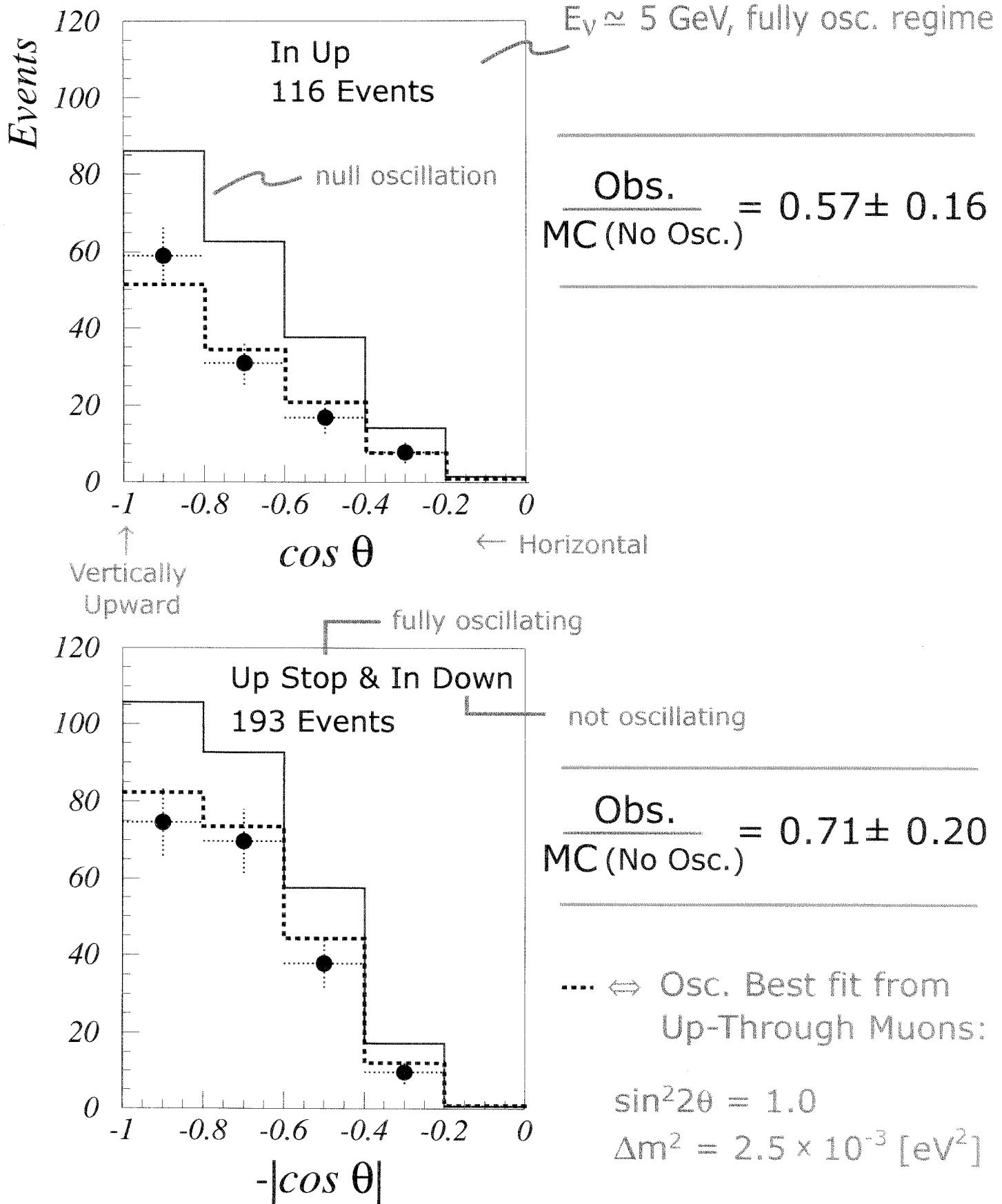


High Energy:

③ Up-Through μ 's
(use scintillator T.O.F.)



MACRO PC Events:



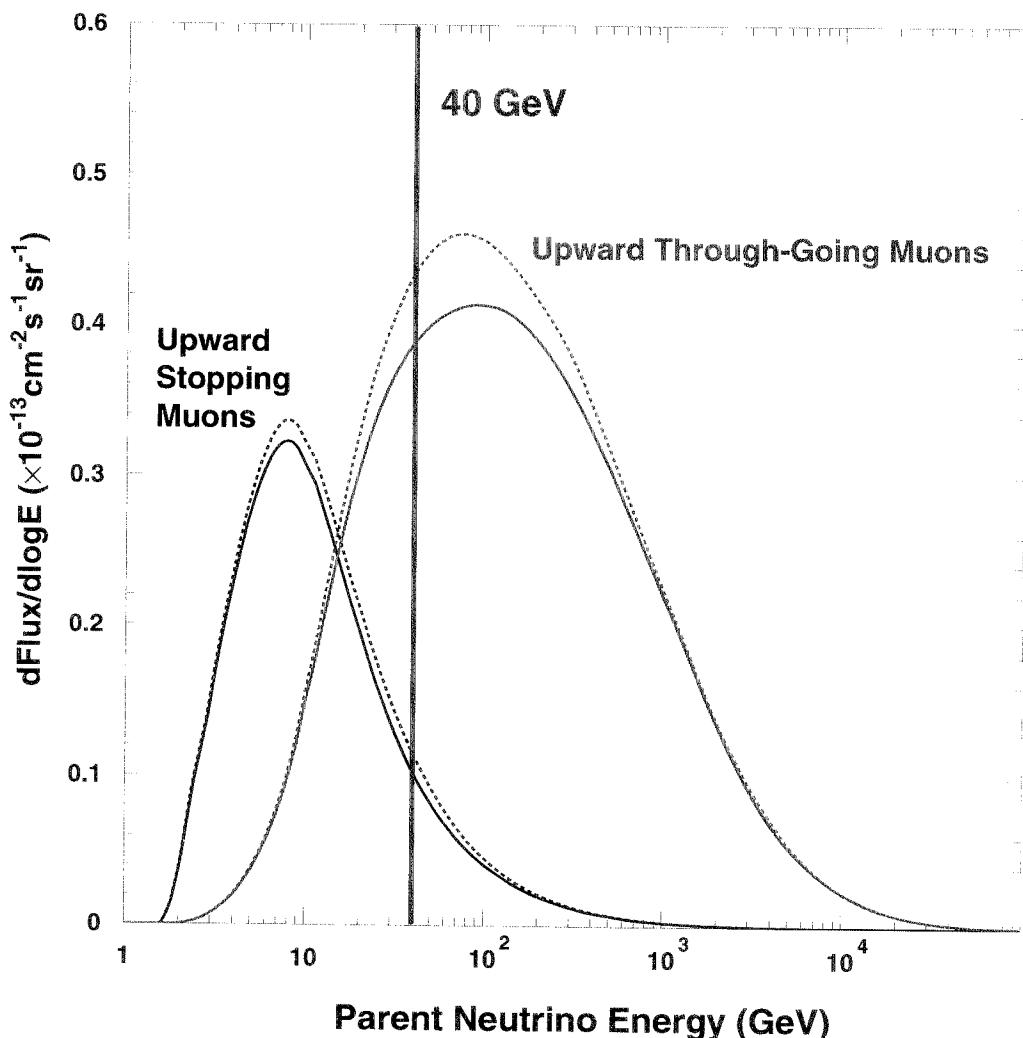
Upward Stopping and Through-going Muons:

$$P(\nu_\mu \rightarrow \nu_\tau) \propto \sin^2\left(\frac{\pi \cdot L}{L_0}\right)$$

$$\text{where } L_0 = \pi \left(\frac{1.27 \cdot \Delta m^2}{E_\nu} \right)^{-1} = 2.47 \cdot \frac{E_\nu}{\Delta m^2}$$

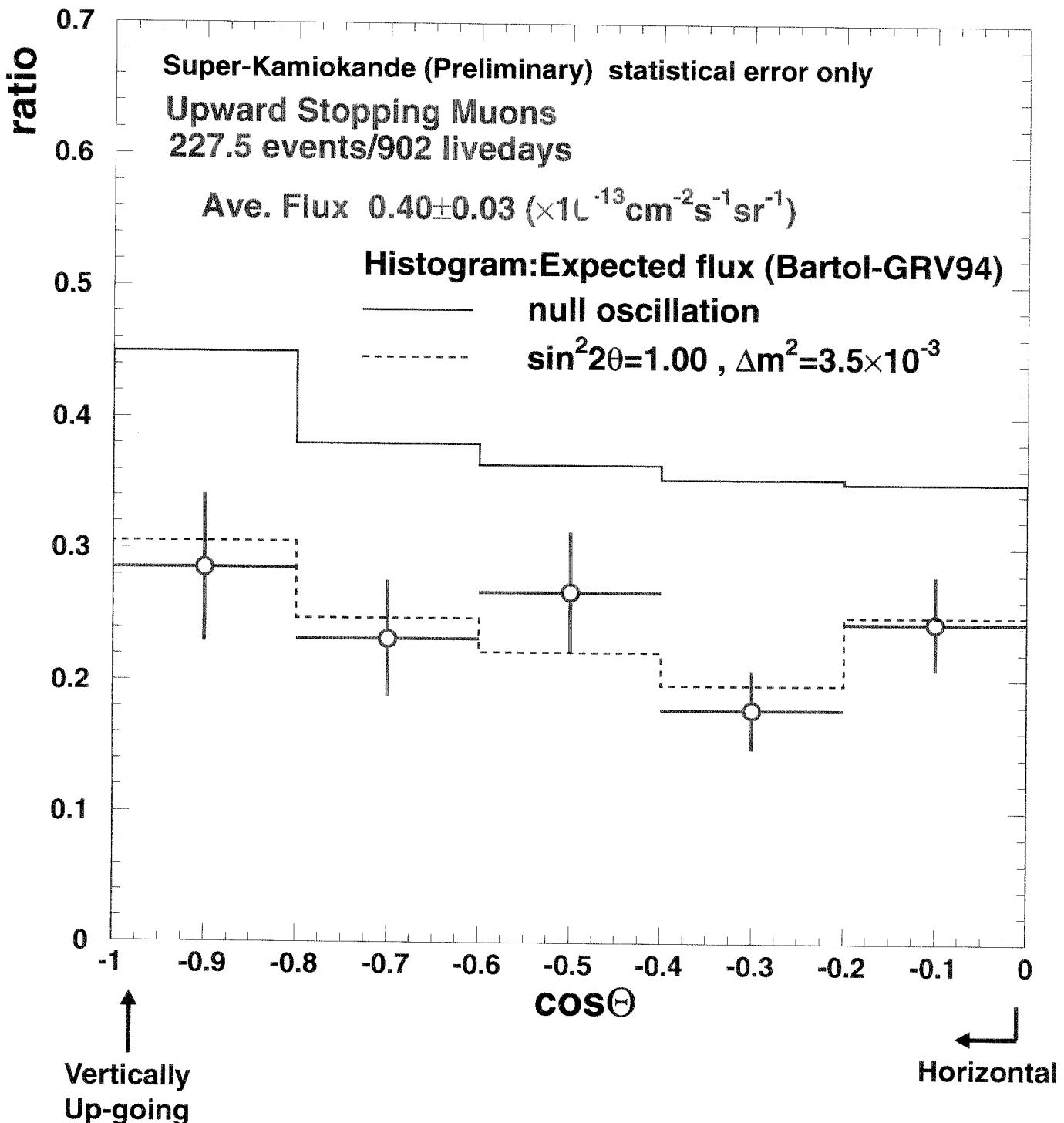
Consider $\Delta m^2 \simeq 5 \times 10^{-3} [\text{eV}^2]$ and $E_\nu = 40 \text{ GeV}$
 then $L_0 \simeq 1.5 \text{ Earth diameters.}$

So, for $E_\nu \gg 40 \text{ GeV} \Rightarrow \text{No oscillations,}$
 however $E_\nu \simeq 40 \text{ GeV} \Rightarrow \text{oscillations.}$



If $\nu_\mu \rightarrow \nu_\tau$, expect $\frac{(\text{stop/thru})_{\text{Data}}}{(\text{stop/thru})_{\text{MC (Null osc.)}}} < 1$

Upward_Stop/Thru Ratio vs. Zenith:



$$r\left(\frac{\text{stop}}{\text{thru}}\right) = \begin{cases} 0.37 \pm 0.05 & (\text{Null Osc.}) \\ 0.24 \pm 0.02 & \text{Observed} \leftrightarrow 2.8 \sigma \text{ low} \end{cases}$$

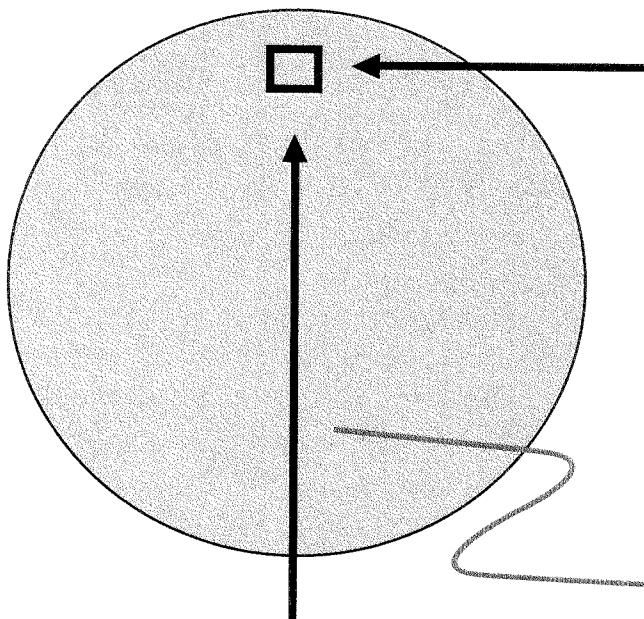
Upward Through-going Muons:

$$\langle E_\nu \rangle_{\text{thru } \mu} \sim 100 \text{ GeV}$$

For $E_\nu = 100 \text{ GeV}$ and $\Delta m^2 \approx 5 \times 10^{-3} [\text{eV}^2]$,

$L_0 \simeq 1.2\pi$ Earth diameters, i.e.

$$P(\nu_\mu \rightarrow \nu_\tau) \propto \sin^2\left(\frac{L[\text{Earth diam.}]}{1.2}\right)$$



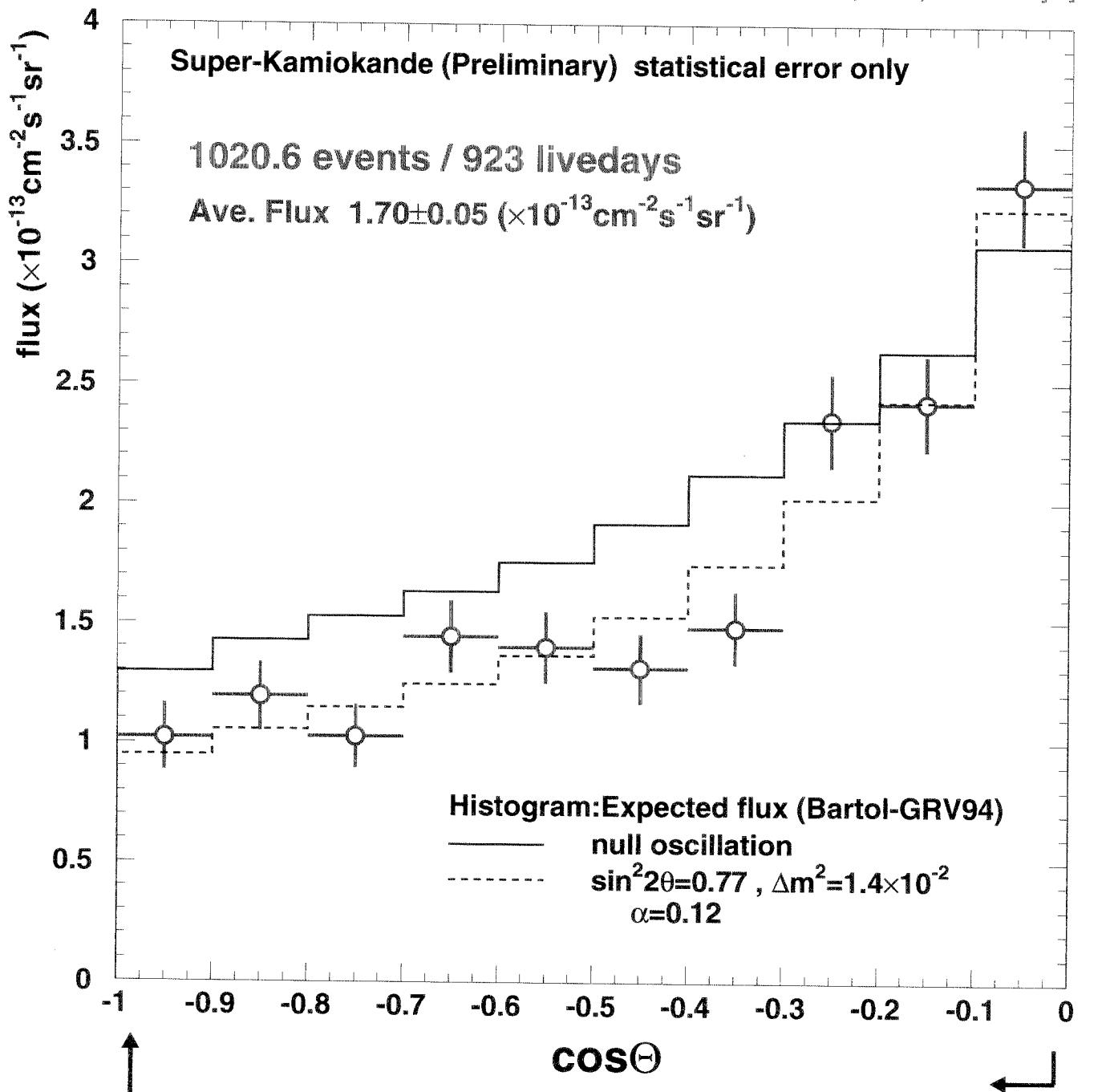
Horizontal muons \Leftrightarrow
 $L \sim 500 \text{ km} \simeq 0.04 \text{ diam.}$
 \Rightarrow No Oscillations

Vertical muons \Leftrightarrow
 $L \sim 13,000 \text{ km} = 1.0 \text{ diam.}$
 \Rightarrow Oscillations

For $E_\nu > 100 \text{ GeV}$, oscillations will also occur for μ incident away from the vertical.

Upward Through-going Muon Flux:

[Phys. Rev. Lett. 82, 2644 (1999) - 537 days]



Vertically
Up-going

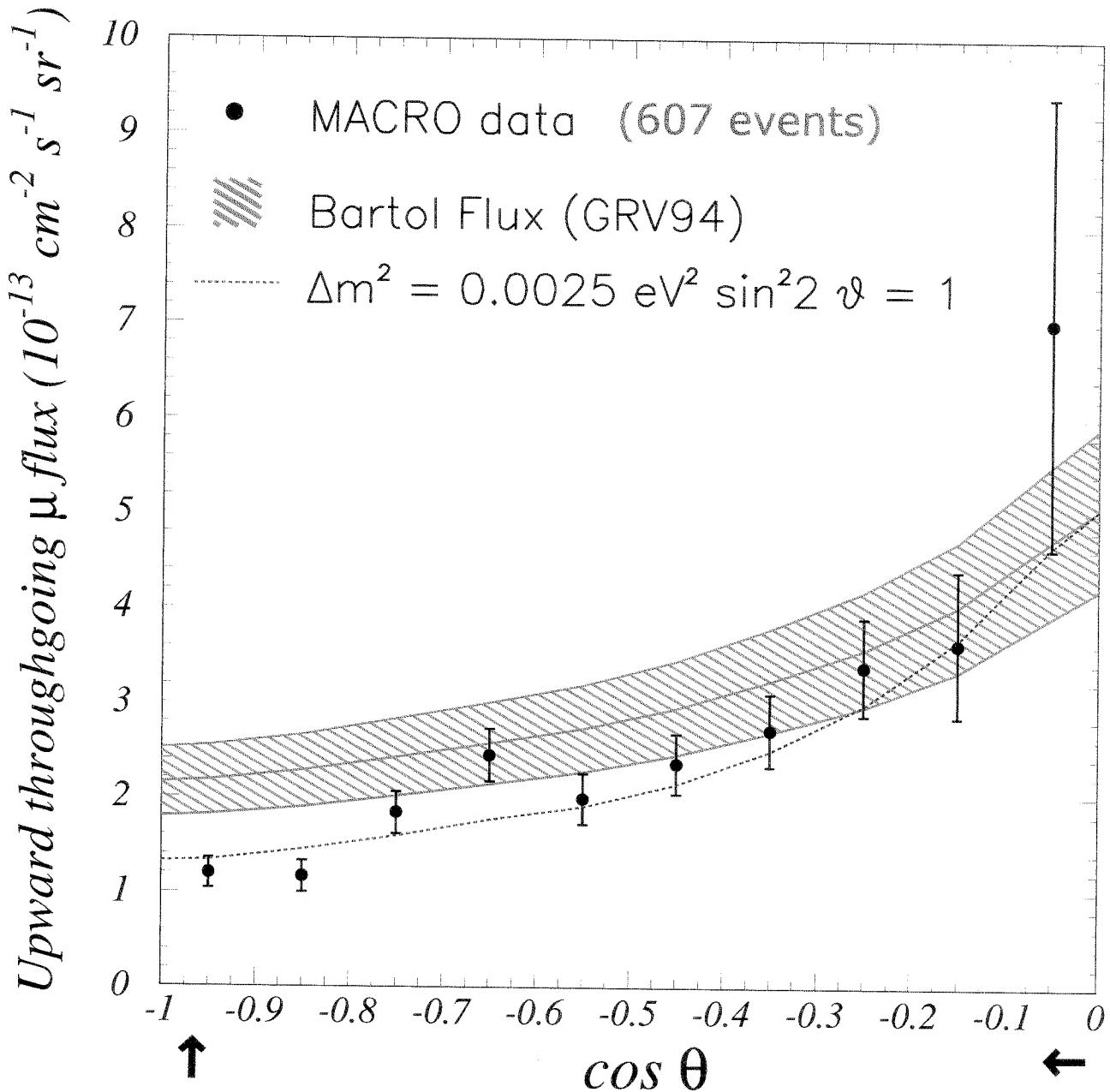
$$\Phi^{\text{data}} = 1.70 \pm 0.05$$

Horizontal

$$\Phi_{\text{no osc.}}^{\text{theory}} = \begin{cases} 1.97 \pm 0.44 & (\text{BARTOL}) \\ 1.84 \pm 0.41 & (\text{Honda}) \end{cases}$$

Angular Distribution of Up-Through Muons:

$E_\mu > 1 \text{ GeV}, \langle E_\nu \rangle \approx 80 \text{ GeV}$



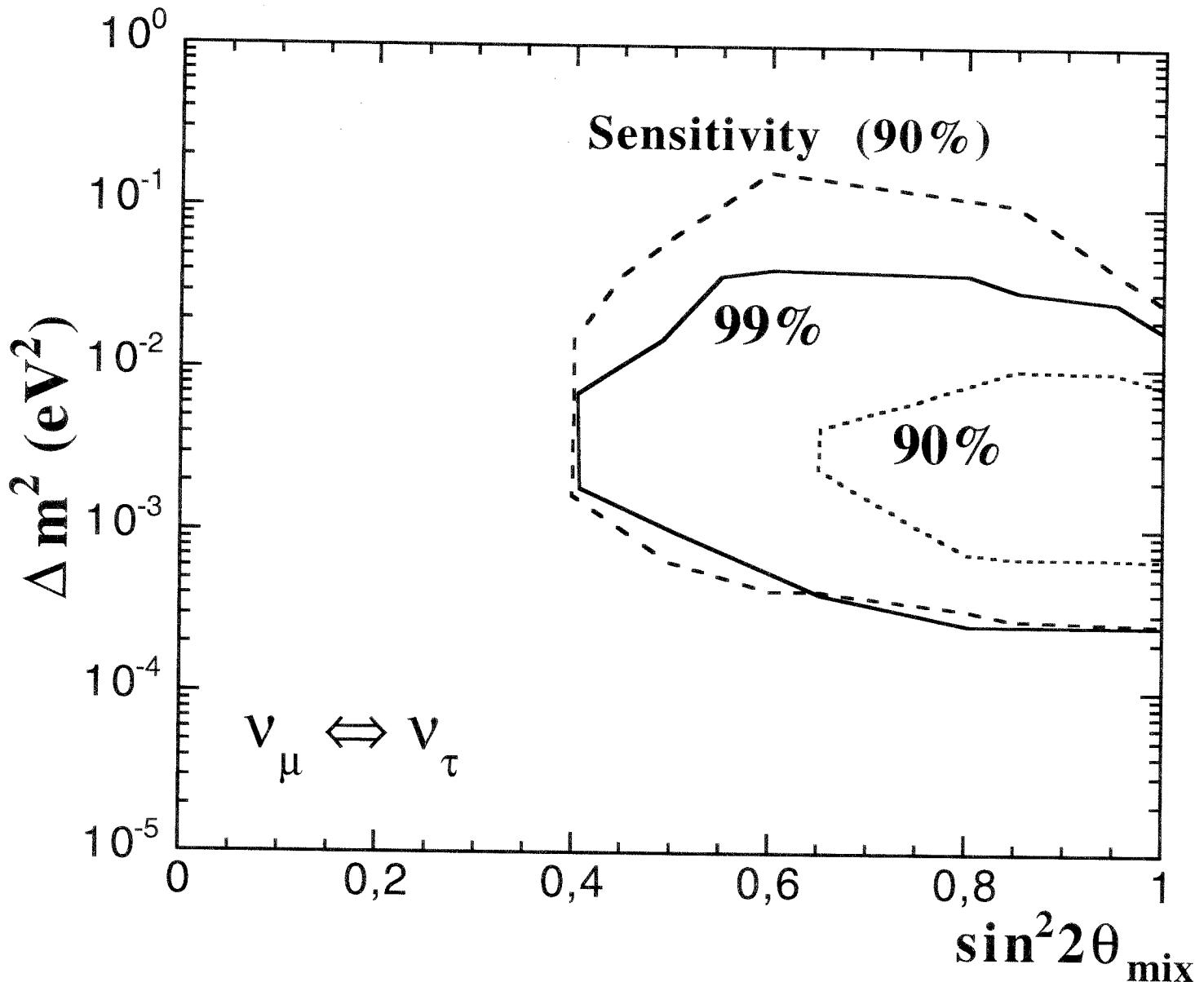
Null Oscillation (norm. free): $\chi^2 = 22.9/8 \text{ d.o.f.}$

$\nu_\mu \leftrightarrow \nu_\tau$ (norm. free): $\chi^2 = 12.5/8 \text{ d.o.f.}$

Data/MC (Null Osc.) = $0.74 \pm 0.03 \pm 0.04 \pm 0.12$
stats. syst. theory

MACRO Allowed Region:

(Feldman-Cousins Method)



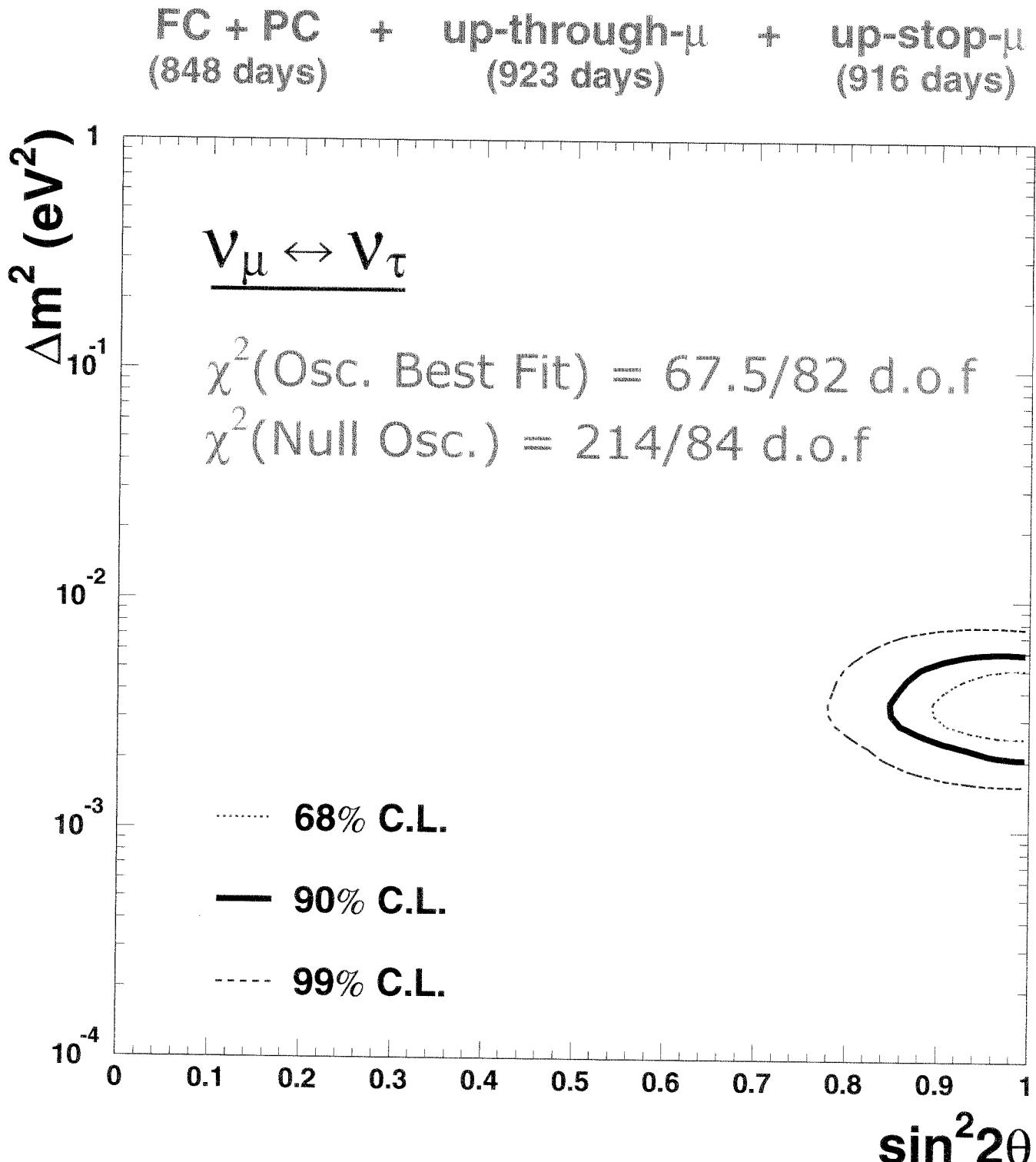
Best Fit:

$$\sin^2 2\theta = 1.0$$

$$\Delta m^2 = 2.5 \times 10^{-3} \text{ [eV}^2\text{]}$$

$$(\chi^2 = 12.5; \chi^2 = 10.6 \text{ at } \sin^2 2\theta = 1.5)$$

SuperK "All Data" Allowed Region:

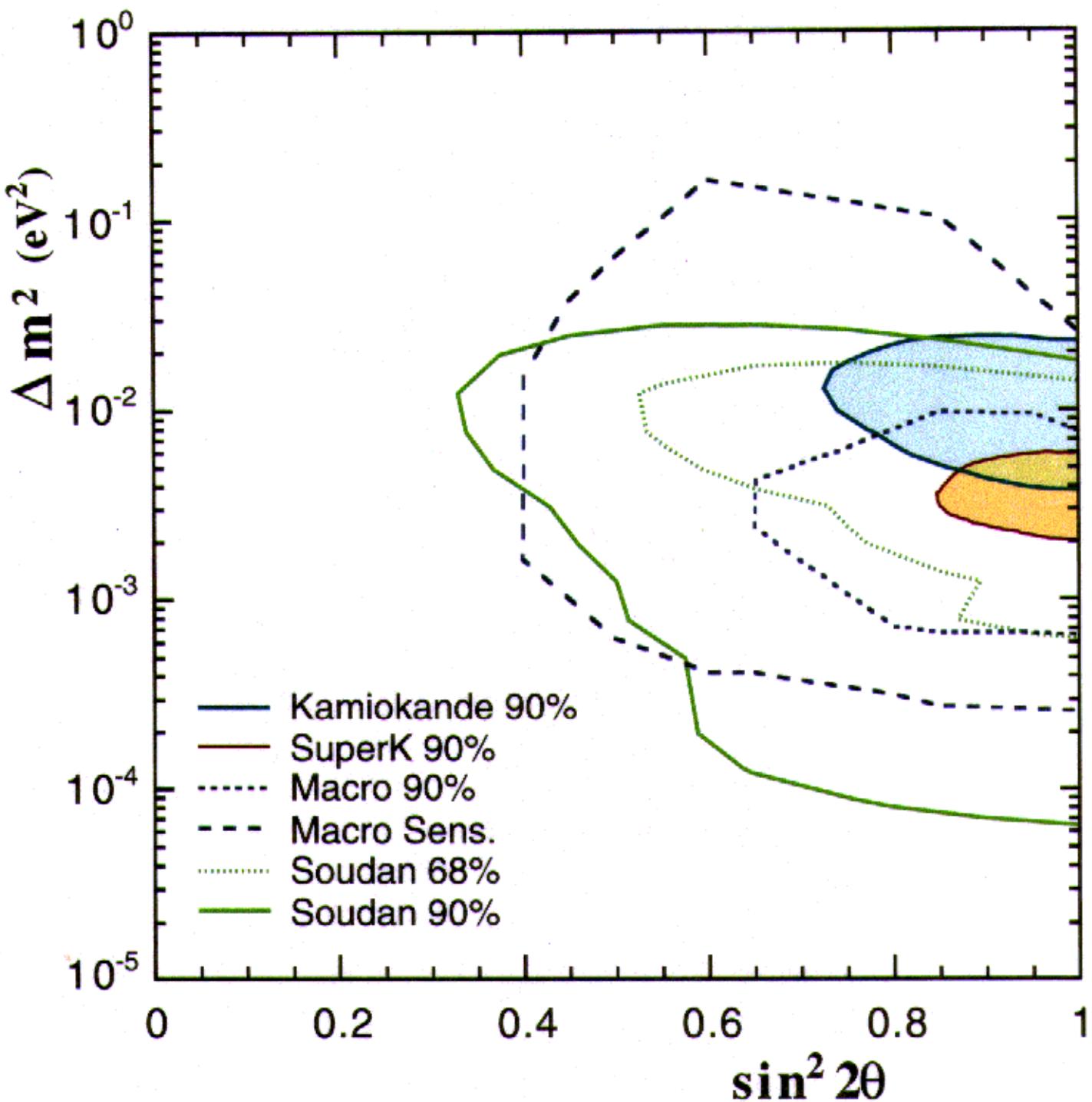


$$\sin^2 2\theta = 1.0$$

$$\Delta m^2 = 3.5 \times 10^{-3} [\text{eV}^2]$$

$$\alpha = 0.06$$

Oscillation Parameters - Best Fits:



$\nu_\mu \leftrightarrow \nu_x$ with large $\sin^2 2\theta, \Delta m^2 \sim 10^{-2} - 10^{-3}$:

What flavors are involved

- in the dominant oscillation?
- in sub-dominant oscillations?

Dominant $\nu_\mu \rightarrow \nu_e$?

Ruled out by the CHOOZ Reactor Exp't

Limit on $\bar{\nu}_e \rightarrow \nu_x \xrightarrow{\text{CPT}} \nu_e \rightarrow \nu_x$ limit.
(hep-ex/9907073)

Dominant $\nu_\mu \rightarrow \nu_\tau$?

Where are the ν_τ events?

Expect ~ 0.9 CC(ν_τ) events per kiloton year

\Rightarrow SuperK has ~ 47 FC or PC CC(ν_τ)'s,

Soudan has ~ 4 FC or PC CC(ν_τ)'s.

These will be upgoing - and practically
indistinguishable from ordinary NC events.

Dominant $\nu_\mu \rightarrow \nu_{\text{sterile}}$?

ν_{sterile} do not exchange Z^0 's with (ordinary) matter:

$\Rightarrow 1) \nu_s N \rightarrow \nu_s N \pi^0$

2) Effective potentials in matter:

$$V_\mu - V_s = \mp \sqrt{2} G_F \frac{N_n}{2}$$

Allowed Region from FC Single-Ring Events:

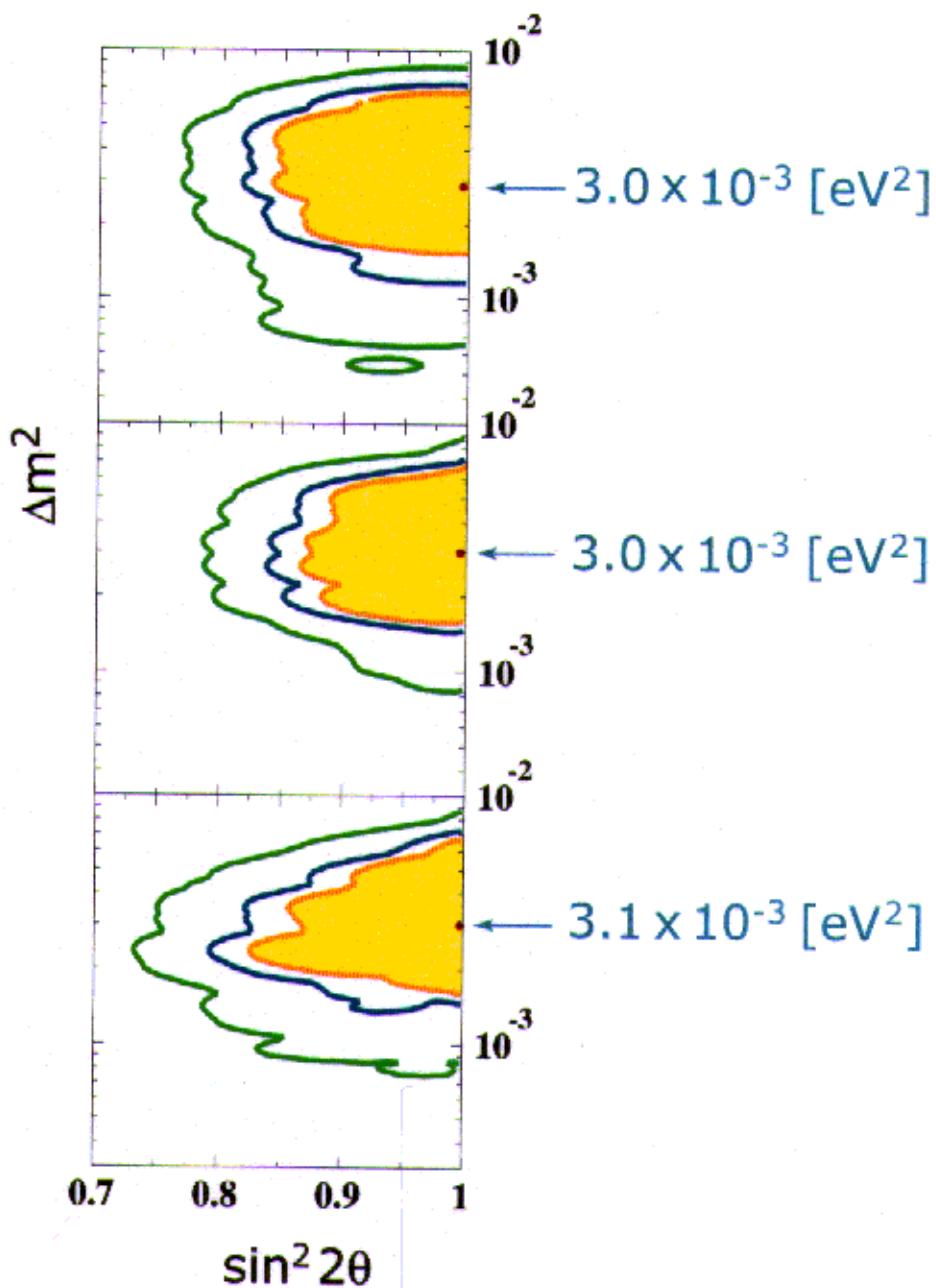
SuperKamiokande (very preliminary)

$\nu_\mu \rightarrow \nu_\tau$

$\nu_\mu \rightarrow \nu_s$
 $\Delta m^2 > 0$

$\nu_\mu \rightarrow \nu_s$
 $\Delta m^2 < 0$

$^{68}_{90}$ C.L.
 $^{90}_{95}$ C.L.
 $^{99}_{99}$ C.L.



Neutrino Oscillation Parameters in Matter:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \cdot \sin^2 \left(\pi \frac{L}{L_0} \right)$$

\downarrow

$$\sin^2 2\theta_m$$

\downarrow

(in vacuum)

(in matter)

\downarrow

L_m

where

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\sin^2 2\theta + (D - \cos 2\theta)^2},$$

$$L_m = \frac{L_0}{[\sin^2 2\theta + (D - \cos 2\theta)^2]^{1/2}},$$

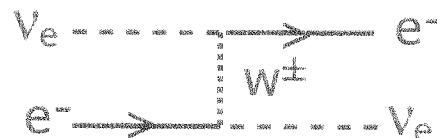
and

$$D = \frac{2E_\nu V_{\alpha\beta}}{\Delta m^2}, \quad V_{\alpha\beta} \equiv V_\alpha - V_\beta$$

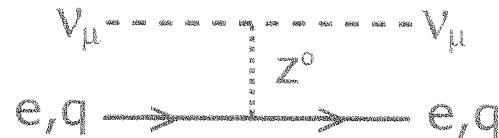
the effective potentials
in matter for ν_α and ν_β .

$$V_{\mu\tau} = 0 \Rightarrow D = 0 \Rightarrow \text{No matter effects.}$$

$$V_{e\mu} = \pm \sqrt{2} G_F N_e$$



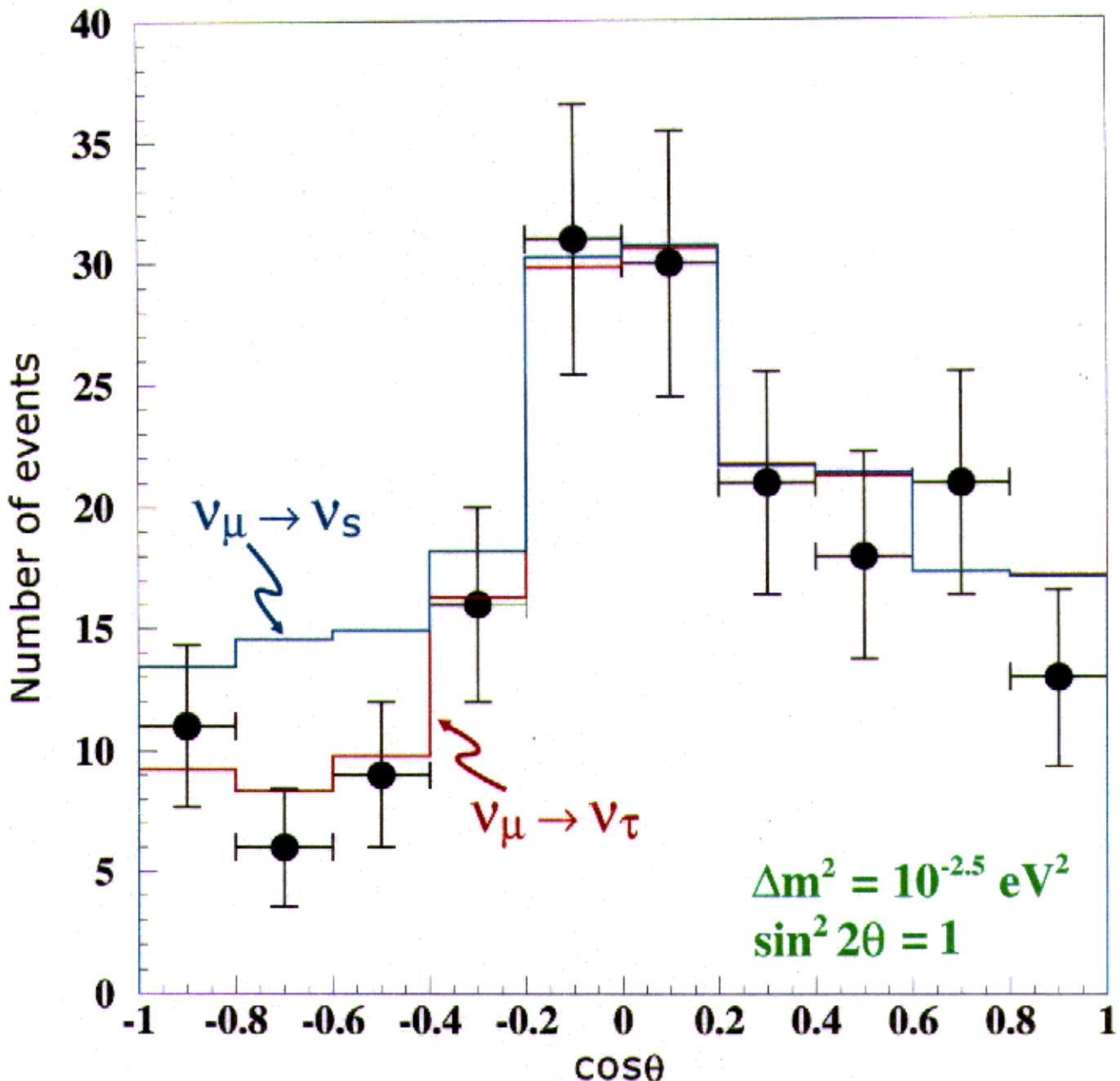
$$V_{\mu s} = \mp \sqrt{2} G_F \frac{N_n}{2}$$



For $D \gg 1$ (high neutrino energies), matter effects will suppress $\nu_\mu \rightarrow \nu_s$ but not $\nu_\mu \rightarrow \nu_\tau$.

Zenith Angle Distribution of PC Events with $N(\text{p.e.}) > 45,000$:

$E_{\text{vis}} > 5 \text{ GeV}, \langle E_V \rangle \sim 25 \text{ GeV}$

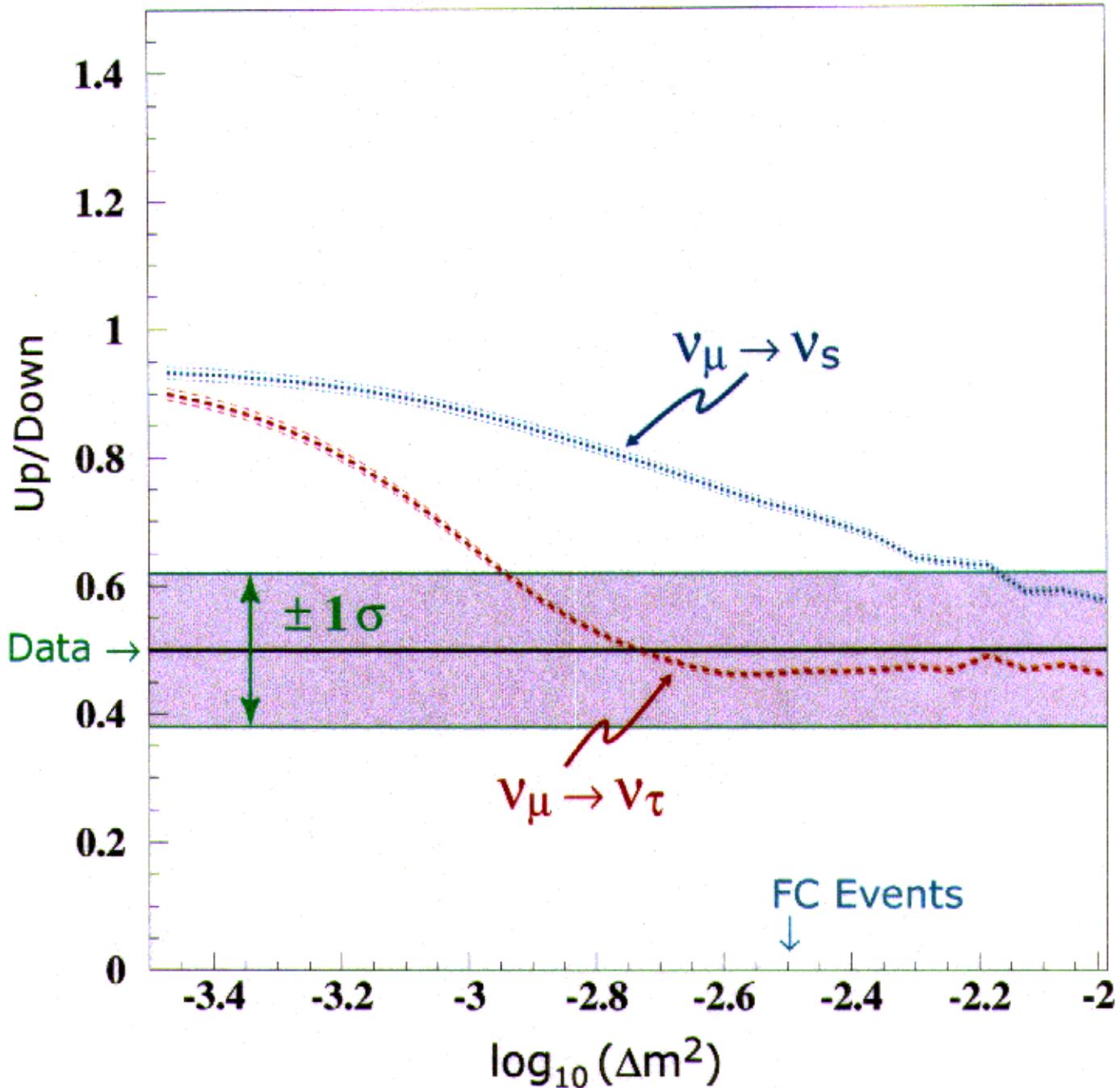


Observe $\frac{N_{\text{up}} (\cos\theta < -0.4)}{N_{\text{down}} (\cos\theta > +0.4)} = 0.50 \pm 0.12 \pm 0.01 \text{ (data)}$

$(0.94 \pm 0.04 \text{ MC, null oscillation.})$

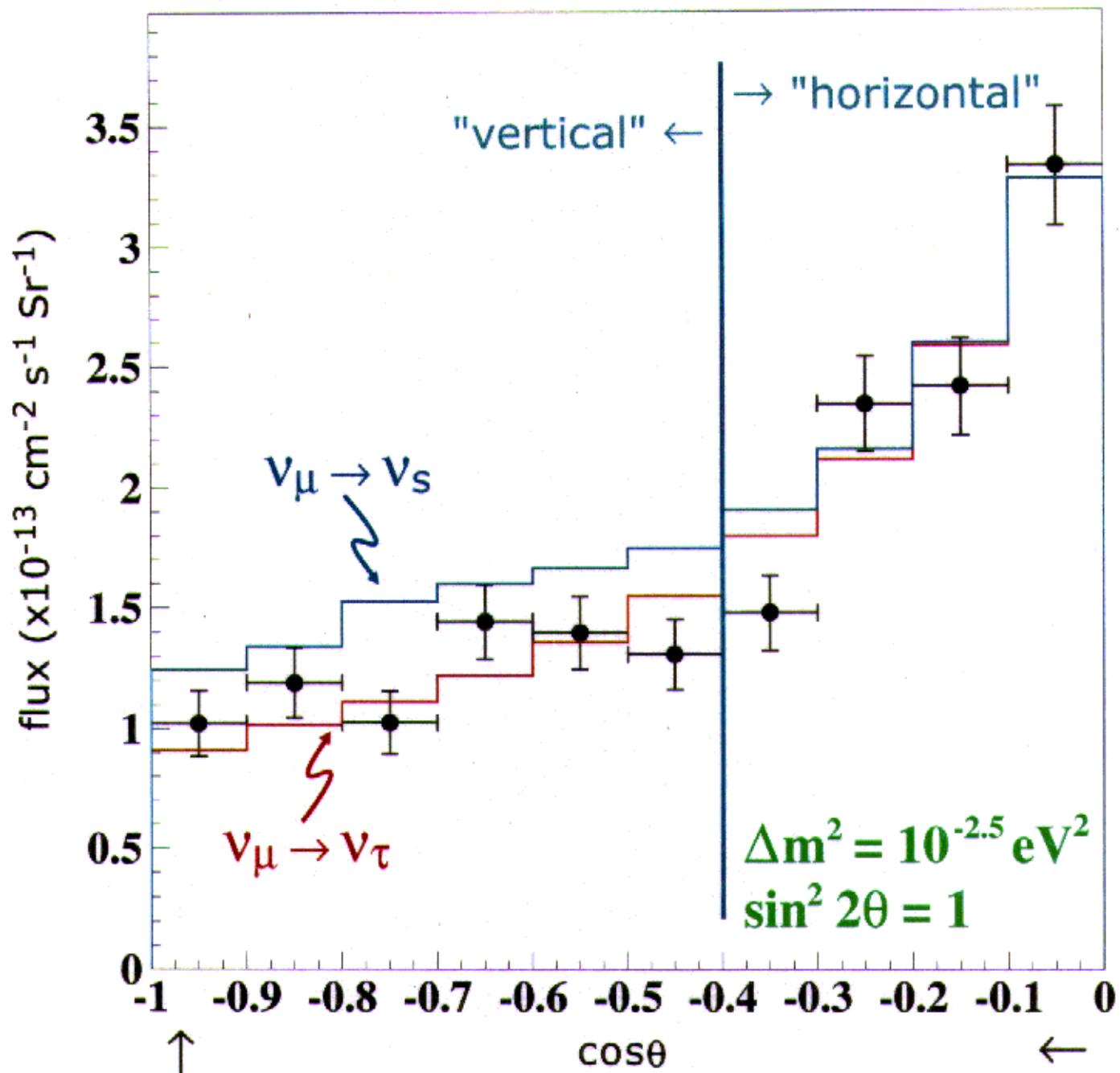
High-energy PC Events:

Up/Down Ratio vs. Δm^2



Define $\chi^2_{PC} = \frac{[(N_{up}/N_{down})_{data} - (N_{up}/N_{down})_{MC}]^2}{\sigma^2}$

Zenith Angle Distribution of Upward Through-going Muons:



Observe $\frac{N_v (\cos\theta < -0.4)}{N_h (\cos\theta > +0.4)} = 0.77 \pm 0.05 \pm 0.04$ (data)

Regions excluded by energetic PC and upward through-going muons:

$$\chi^2 = \chi^2_{\text{PC}} + \chi^2_{\text{thru } \mu}$$

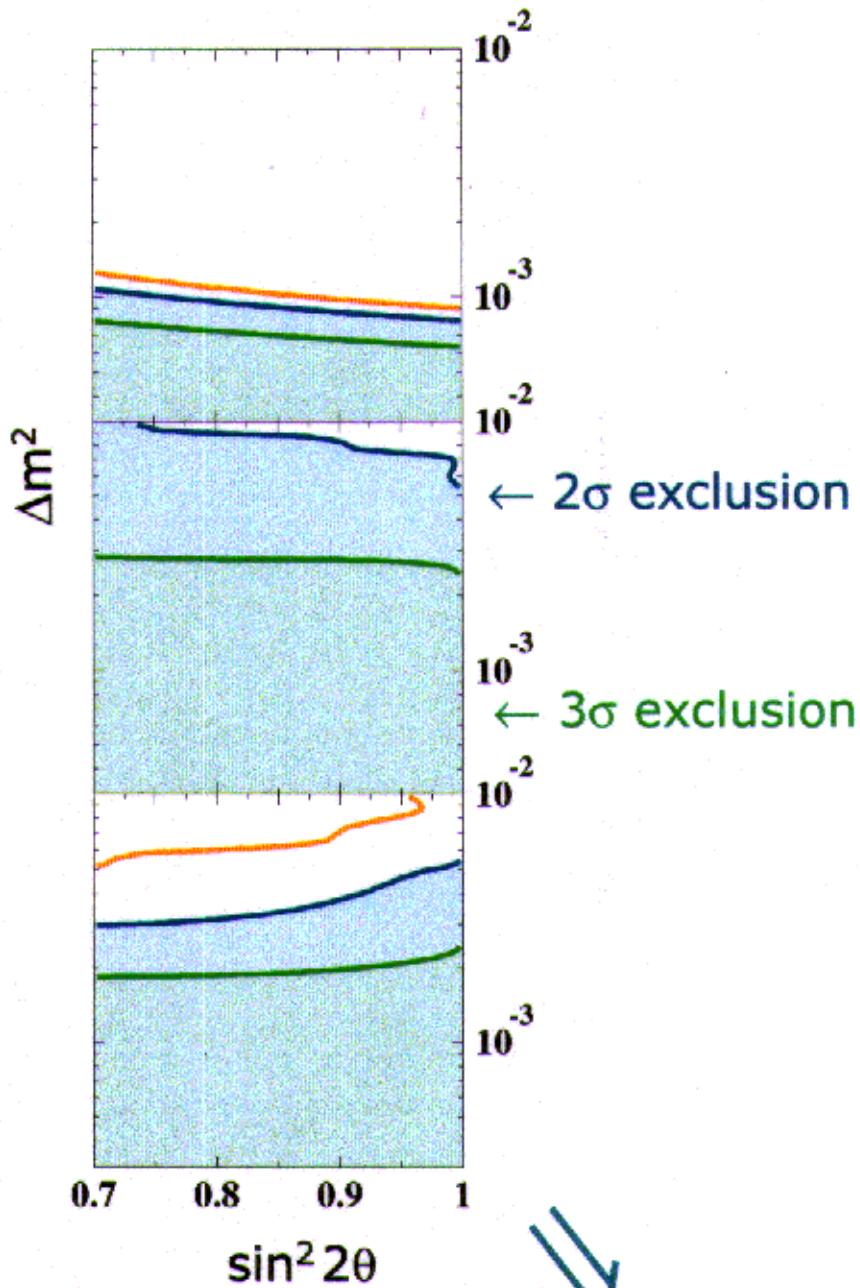
$\nu_\mu \rightarrow \nu_\tau$

$\nu_\mu \rightarrow \nu_s$

$\Delta m^2 > 0$

$\nu_\mu \rightarrow \nu_s$

$\Delta m^2 < 0$



99% C.L.

99% C.L. $\simeq 2\sigma$ exclusion

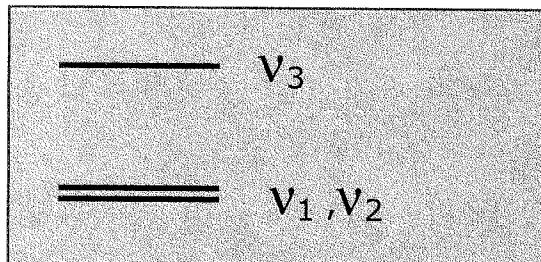
99% C.L. $\simeq 3\sigma$ exclusion

$\nu_\mu \rightarrow \nu_{\text{sterile}}$ disfavored
at $\sim 2\sigma$ level.

Three-Flavor Mixing:

$$\begin{bmatrix} v_e \\ v_\mu \\ v_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Parametrize in the "one-dominant mass scale approximation," by assuming the mass spectrum



$$\left\{ \begin{array}{l} \Delta m^2 = |m_3^2 - m_{1,2}^2| \\ \delta m^2 = |m_2^2 - m_1^2| \text{ (responsible of solar } v \text{ deficit)} \end{array} \right.$$

Up to terms of order $(\delta m^2 / \Delta m^2)$, the parameter space is spanned by

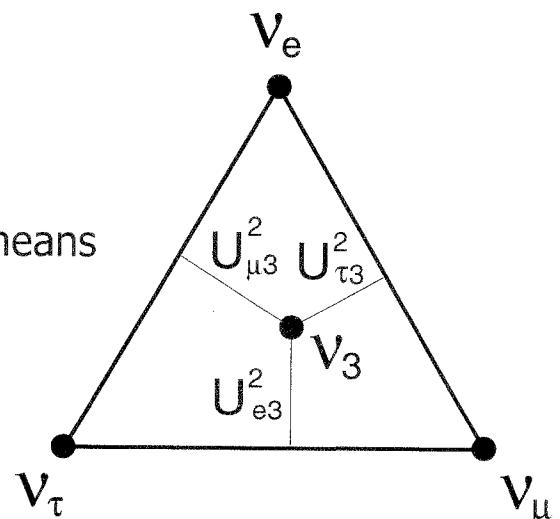
$$(\Delta m^2, U_{e3}^2, U_{\mu 3}^2, U_{\tau 3}^2) \text{ with } U_{e3}^2 + U_{\mu 3}^2 + U_{\tau 3}^2 = 1 \text{ (unitarity)}$$

$$P^{\text{vac}}(v_\alpha \leftrightarrow v_\beta) = 4 U_{\alpha 3}^2 U_{\beta 3}^2 \cdot \sin^2 \left[\frac{1.27 \Delta m^2 \cdot L}{E_v} \right].$$

What is relevant is the flavor composition of v_3 :

$$v_3 = U_{e3} v_e + U_{\mu 3} v_\mu + U_{\tau 3} v_\tau$$

The unitarity being automatically enforced by means of a "triangular representation" for Δm^2 fixed.



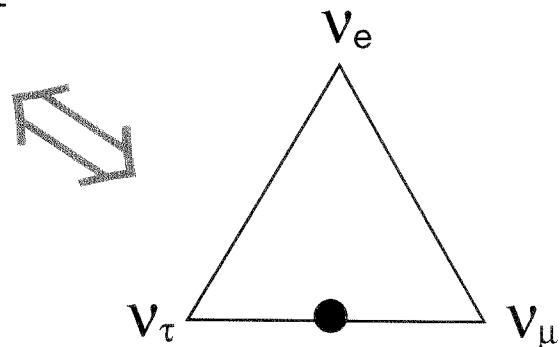
Ref: Fogli, Lisi, Marrone, Scioscia,
 Phys. Rev. D59, 033001(1998); hep-ph/9904465.

Comparing expectations with SK zenith angle distributions
 for specific choices of (Δm^2 , U_{e3}^2 , $U_{\mu 3}^2$, $U_{\tau 3}^2$) and including
 the CHOOZ constraint - two flavor oscillations
 with maximal $V_\mu \leftrightarrow V_\tau$ mixing works rather well.

$$\sin^2 2\theta_{\mu\tau} = 4 U_{\mu 3}^2 U_{\tau 3}^2 \simeq 1.0$$

$$U_{\mu 3}^2 \simeq U_{\tau 3}^2 \simeq \frac{1}{2}$$

$$U_{e3}^2 \simeq 0.$$



However a small admixture of U_{e3}^2 is allowed:

$$U_{e3}^2 \lesssim 0.05$$

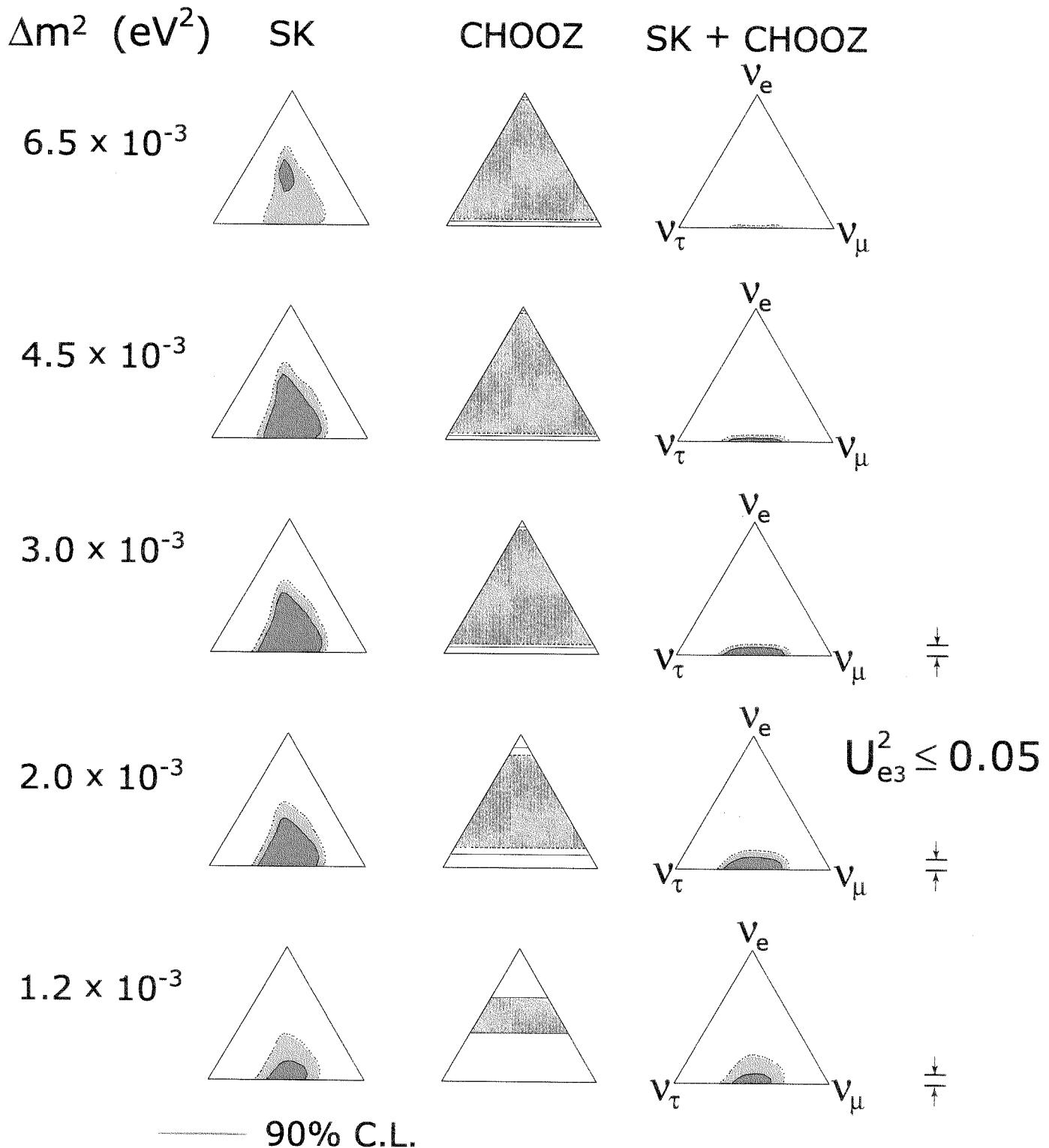
Remark:

In the Bi-Maximal Mixing Model

$$U \approx \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad U_{e3} = 0.$$

Allowed (U_{e3}^2 , $U_{\mu 3}^2$, $U_{\tau 3}^2$) versus Δm^2 :

(Fogli et al.)

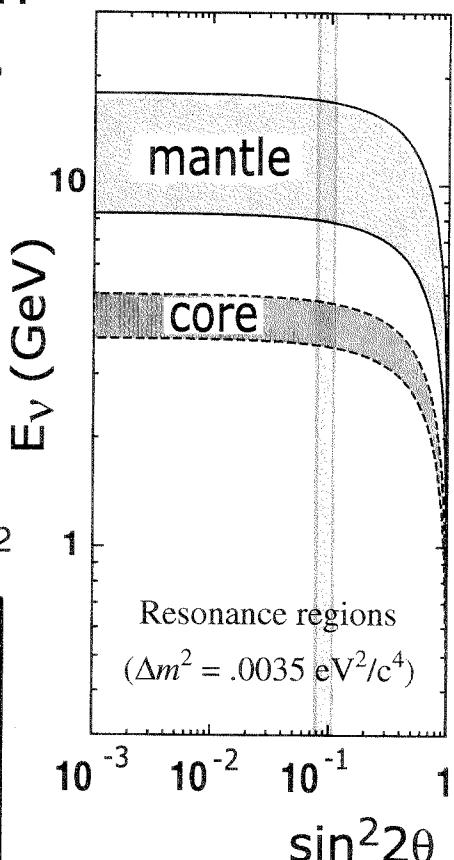


$$P^{\text{vac}}(v_\mu \rightarrow v_e) = \underbrace{4 \cdot U_{\mu 3}^2 \cdot U_{e3}^2}_{\leq 0.10} \cdot \sin^2 \left(\frac{1.27 \cdot \Delta m^2 \cdot L}{E_v} \right)$$

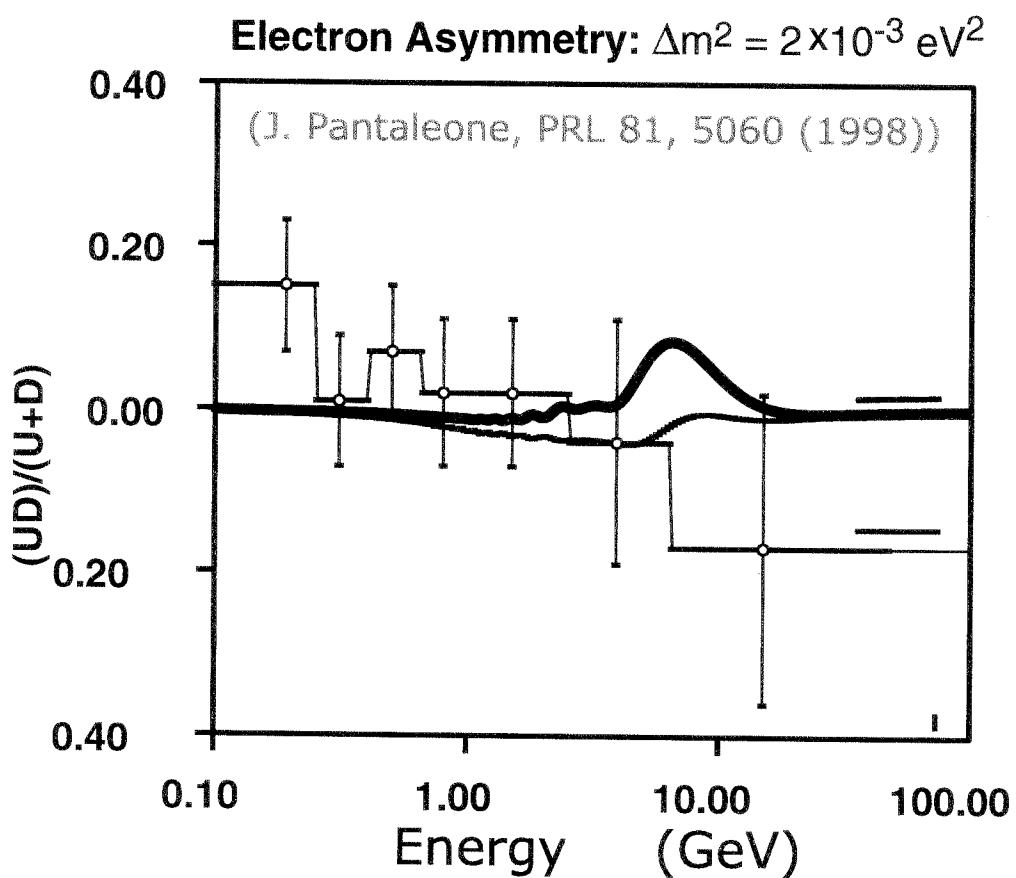
Amplification of $P(\nu_\mu \rightarrow \nu_e)$ via Matter Resonances in the Earth:

I. MSW Resonance - can occur in Mantle and in the Core.

(Barger, Geer, Whisnant:
[hep-ph/9906487](#))



May get ν_e bump at the resonance energy:



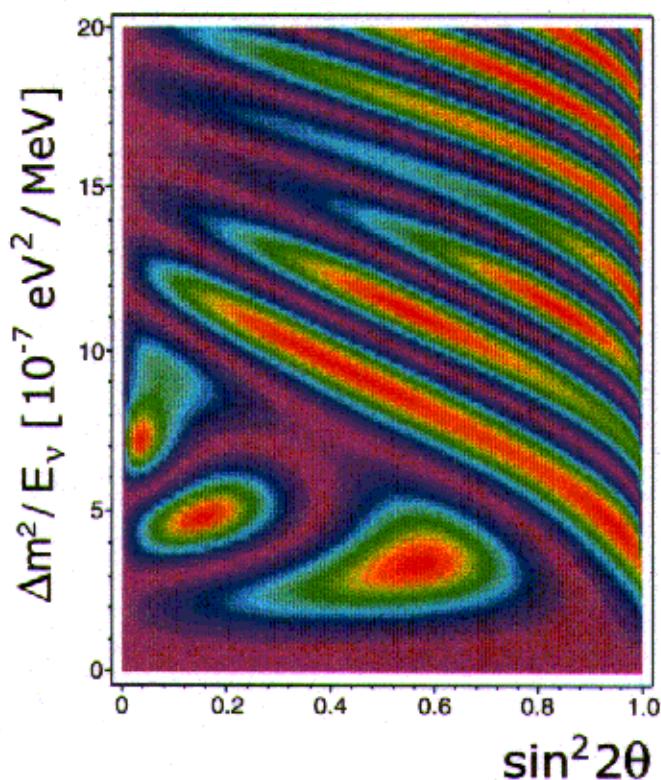
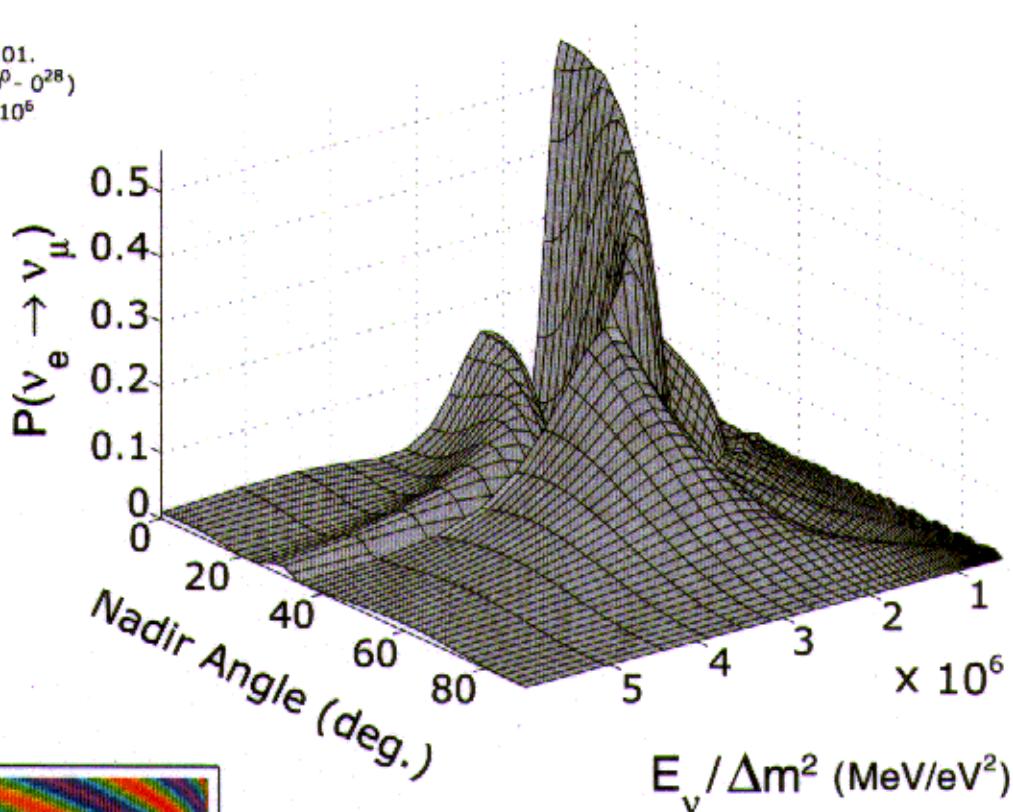
MSW enhancement can also occur in the Earth's crust.
(LBL: P.Lipari, [hep-ph/9903481](#)).

II. A different (\neq MSW) resonance-like enhancement may take place for atm. ν 's which cross the Earth's core (mantle-core-mantle trajectories):

(M.V. Chizhov, S.T. Petcov:
[hep-ph/9903424](#))

$$\sin^2 2\theta_{\text{vac}} = 0.010$$

The Probability $P(\nu_{e(\mu)} \rightarrow \nu_{\mu; \tau(e)})$ as a function of h and $E/\Delta m^2$ for $\sin^2 2\theta = 0.01$.
 The NOLR absolute maximum for $h = (0^0 - 0^{28})$ is clearly seen at $E/\Delta m^2 = (1.3 - 1.6) \times 10^6$ MeV/eV². The local maximum at $E/\Delta m^2 = 2.5 \times 10^6$ MeV/eV² is due to the MSW effect in the Earth mantle.



Integrating over E_ν , $\cos \theta_z$, and detector resolution . . . matter resonance may be hard to observe.

. . . Interesting mission for a ν -factory at a Muon Collider.

Near-term "targets" for experimentation:

1. Precision determination of Δm^2 and $\sin^2 2\theta$:

$\sin^2 2\theta$ - how close to 1.0 ?

$$\Delta m^2 = (?) \times 10^{-3} [\text{eV}^2]$$

2. The dominant $\nu_\mu \rightarrow \nu_x$ oscillation:

Is $x = \text{tau}$ or sterile ?

3. Sub-dominant $\nu_\mu \leftrightarrow \nu_e$:

Does it occur in the atm. ν flux? ($U_{e3}^2 > 0$?)

4. For ν 's traversing the Earth - are oscillations matter-enhanced?

By the MSW effect? By the mantle-core-mantle mechanism (be it NOLR or parametric)?

5. Better quantitative understanding needed, of the absolute rates and shape of the atm. ν fluxes.

And other measurements we'd like to see:

6. Comparison of $\Phi(\bar{\nu})$ versus $\Phi(\nu)$:

e.g. $N(\bar{\nu}_\mu)/N(\nu_\mu)$.

7. Flux correlations with the solar cycle?