TAU PHYSICS

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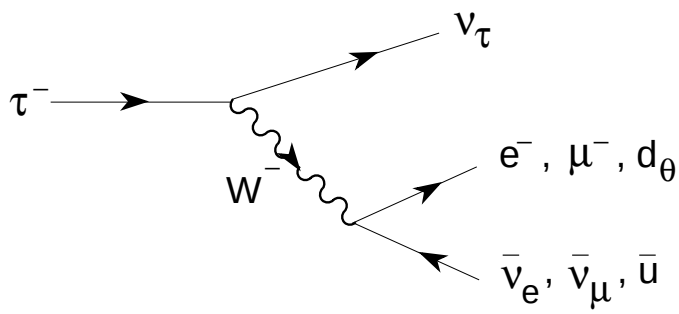
- Lepton Universality
- Lorentz Structure
- Neutral Currents
- The ν_τ Mass
- Lepton Number Violation
- Hadronic Decays
- QCD Tests

A large amount of good experimental work

[Proc. TAU'98, Nucl. Phys. (PS) 76 (1999)]

LP'99, Stanford, August 9–14, 1999

TAU DECAY



$d_\theta \equiv \cos\theta_C\, d + \sin\theta_C\, s$

$$B_l \equiv Br(\tau^- \to \nu_\tau\, l^- \bar{\nu}_l) \approx \frac{1}{5} = 20\%$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau\, e^- \bar{\nu}_e)} \approx N_C = 3$$

$\tau_\tau = (290.77 \pm 0.99)$ fs

$B_e = (17.791 \pm 0.054)\%$; $B_\mu = (17.333 \pm 0.054)\%$

$$R_\tau^B \equiv \frac{1 - B_e - B_\mu}{B_e} = 3.647 \pm 0.014$$

$$R_\tau^\Gamma \equiv \frac{\Gamma_\tau - \Gamma_{\tau\to e} - \Gamma_{\tau\to\mu}}{\Gamma_{\tau\to e}} = 3.640 \pm 0.020$$

$\Rightarrow$ $R_\tau = 3.644 \pm 0.012$

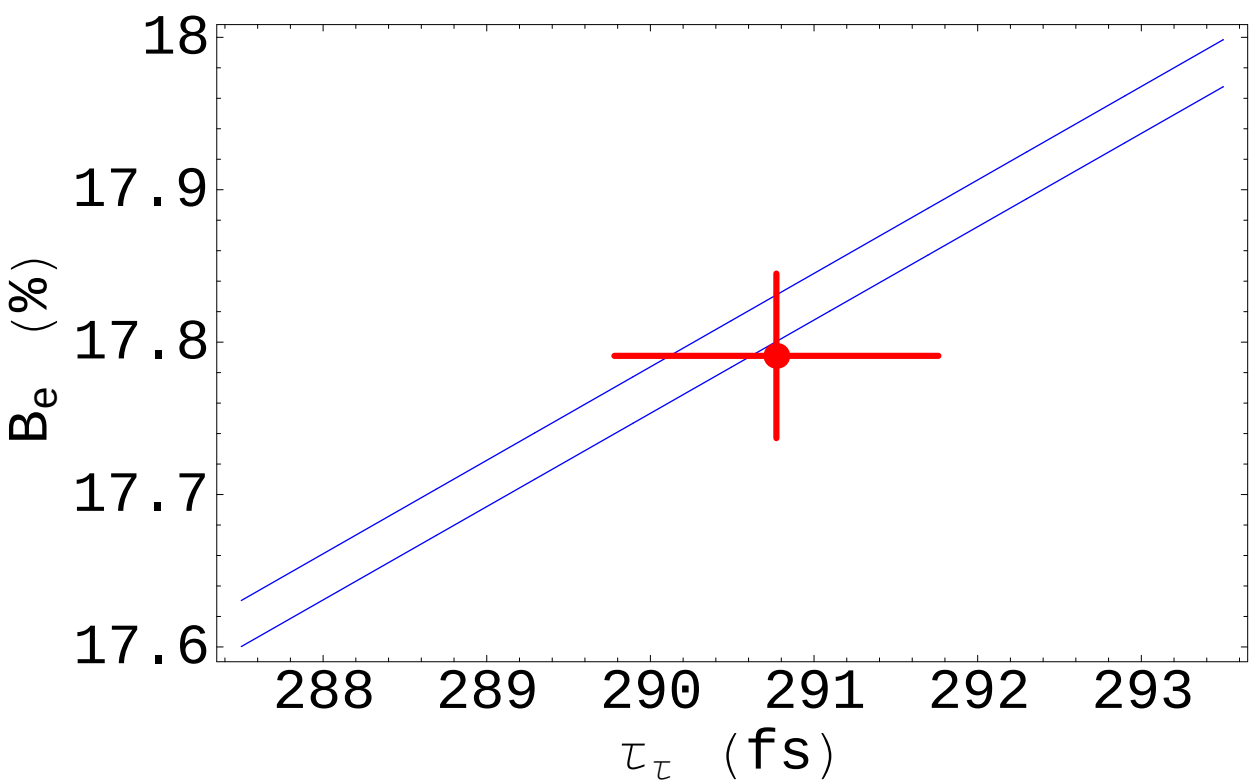
$$\Gamma(\tau^- \to \nu_\tau l^- \bar{\nu}_\tau) = \frac{G_F^2\, m_\tau^5}{192\,\pi^3}\; f(m_l^2/m_\tau^2)\; r_{EW}$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \qquad r_{EW} \approx 0.996$$

$$m_\tau = 1777.05\,^{+0.29}_{-0.26}\ \text{MeV}, \qquad G_F = 1.16637\,(1)\ \times 10^{-5}\ \text{GeV}^{-2}$$

$$\Downarrow$$

$$B_e = \frac{B_\mu}{0.972564 \pm 0.000010} = \frac{\tau_\tau}{(1632.1 \pm 1.4) \times 10^{-15}\ \text{S}}$$



CHARGED–CURRENT UNIVERSALITY

	$\lvert g_\mu/g_e \rvert$
$B_{\tau\to\mu}/B_{\tau\to e}$	1.0009 ± 0.0022
$B_{\pi\to e}/B_{\pi\to\mu}$	1.0017 ± 0.0015
$\sigma \cdot B_{W\to\mu/e}$ $(p\bar{p})$	0.98 ± 0.03
$B_{W\to\mu/e}$ (LEP2)	1.002 ± 0.016

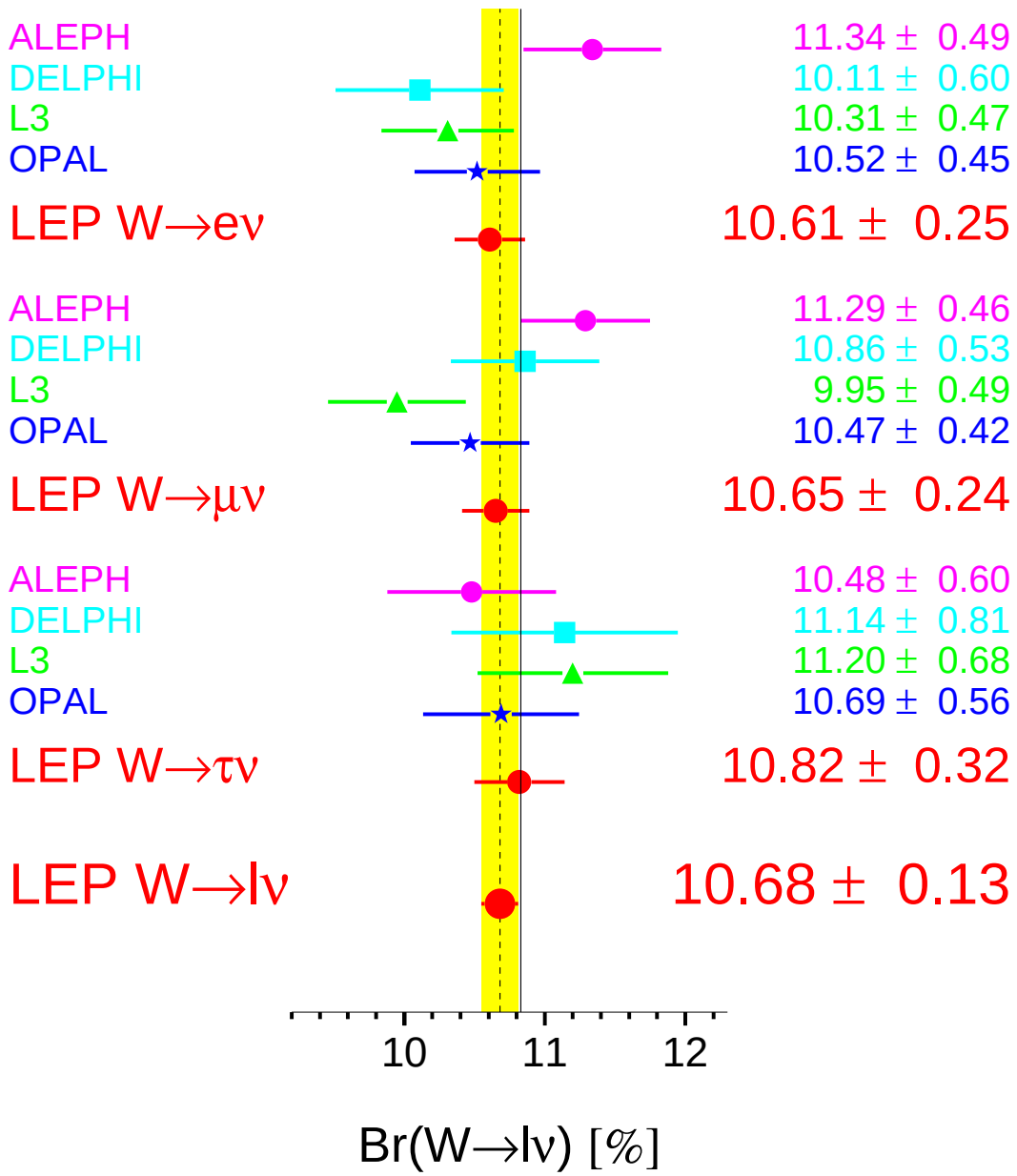
	$\lvert g_\tau/g_\mu \rvert$
$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	0.9993 ± 0.0023
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	1.005 ± 0.005
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.981 ± 0.018
$B_{W\to\tau/\mu}$ (LEP2)	1.008 ± 0.019

	$\lvert g_\tau/g_e \rvert$
$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0002 ± 0.0023
$\sigma \cdot B_{W\to\tau/e}$ $(p\bar{p})$	0.987 ± 0.025
$B_{W\to\tau/e}$ (LEP2)	1.010 ± 0.019

05/07/1999

Tampere 99 - Preliminary - [161-189] GeV

W Leptonic Branching Ratios



CDF + D0: $B(W \to e\nu) = (10.43 \pm 0.17)\%$

LORENTZ STRUCTURE

$l^- \to \nu_l \, l'^- \bar{\nu}_{l'}$ $\qquad$ $(\mu \to e, \ \tau \to \mu, \ \tau \to e)$

$$\mathcal{H} = 4 \frac{G_{l'l}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\,\omega} \left[\overline{l'_\epsilon} \Gamma^n (\nu_{l'})_\sigma\right] \left[\overline{(\nu_l)_\lambda} \Gamma_n \, l_\omega\right]$$

$\Gamma^S = I \quad ; \quad \Gamma^V = \gamma^\mu \quad ; \quad \Gamma^T = \sigma^{\mu\nu}/\sqrt{2} \quad ; \quad \epsilon, \omega, \sigma, \lambda = L, R$

10 complex couplings $g^n_{\epsilon\,\omega}$ for each decay

3×19 real parameters (1 arbitrary phase)

Normalization:

$$\Gamma \propto \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2\right) + 3\left(|g^T_{RL}|^2 + |g^T_{LR}|^2\right) + \left(|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^V_{LL}|^2\right)$$

$$\equiv 1 \equiv Q_{LL} + Q_{LR} + Q_{RL} + Q_{RR}$$

Standard Model: $\qquad G_{l'l} = G_F \quad ; \quad g^V_{LL} = 1$

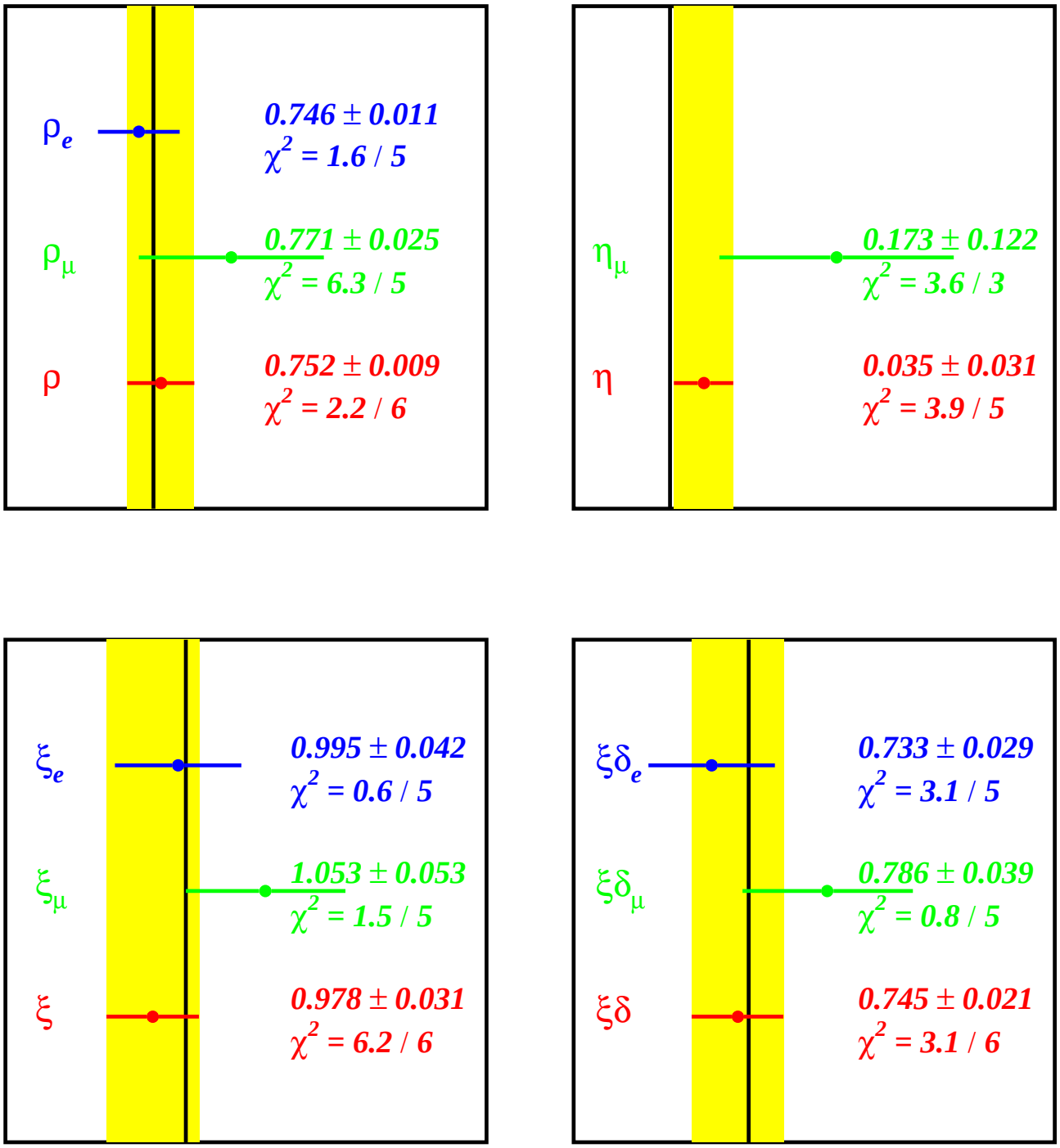
l' Spectrum: $\qquad$ Michel Parameters $\qquad (\delta, \eta, \xi, \delta)$

$$Q_{l_R} \equiv Q_{RR} + Q_{LR} = \frac{1}{2}\left[1 + \frac{\xi}{3} - \frac{16}{9}\xi\,\delta\right]$$

MICHEL PARAMETERS

World Averages (I. Boyko)

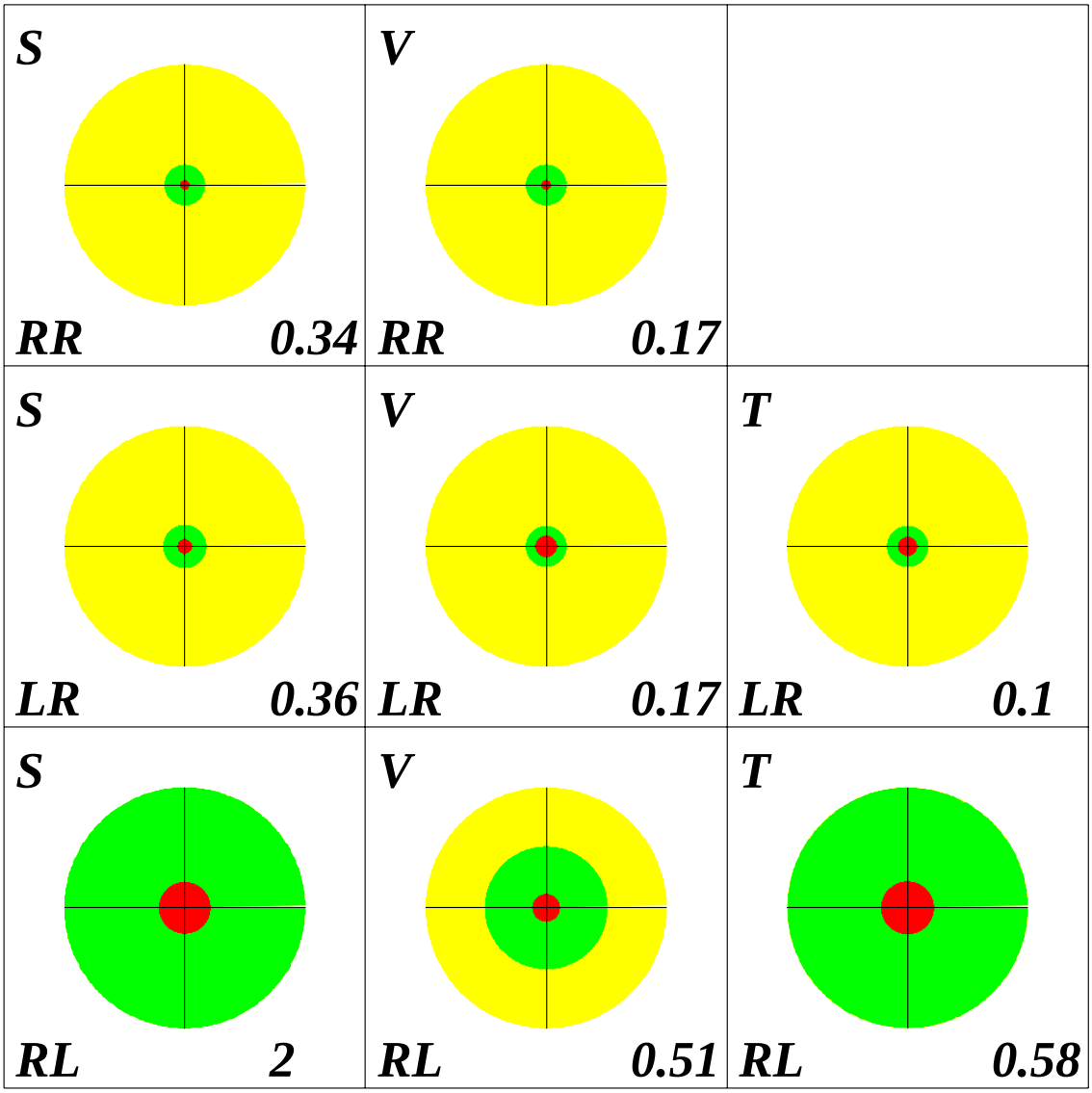
ALEPH, ARGUS, CLEO, DELPHI, L3, OPAL, SLD



Upper Limits on $\tilde{g}^n_{\epsilon\omega} \equiv g^n_{\epsilon\omega}/N^n$ (I. Boyko)

$N^n \equiv \max(g^n_{\epsilon\omega})$ $[N^S = 2\,,\ N^V = 1\,,\ N^T = 1/\sqrt{3}]$

e/μ Universality assumed



(90% CL) Red circles = μ–decay limits

$Q_{\tau_R} < 0.032$ $\left[Q^{\tau\to e}_{\tau_R} < 0.054\ ;\ Q^{\tau\to\mu}_{\tau_R} < 0.047\right]$

$\tau^- \to \nu_\tau l^- \bar{\nu}_l \gamma$

CLEO ($E_\gamma > 10$ MeV)

	Data	MC
$B_{\mu\gamma}$ ($\times 10^{-3}$)	$3.61 \pm 0.16 \pm 0.35$	3.68 ± 0.02
$B_{e\gamma}$ ($\times 10^{-2}$)	$1.75 \pm 0.06 \pm 0.17$	1.86 ± 0.01
$B_{e\gamma}/B_{\mu\gamma}$	$4.85 \pm 0.27 \pm 0.57$	5.05 ± 0.04

Good agreement with Standard Model predictions

First direct measurement of $Br(\tau^- \to \nu_\tau e^- \bar{\nu}_e \gamma)$

For $E_\gamma > 20$ MeV:

OPAL 96 : $B_{\mu\gamma} = (3.0 \pm 0.4 \pm 0.5) \times 10^{-3}$

CLEO 99 : $B_{\mu\gamma} = (3.04 \pm 0.14 \pm 0.30) \times 10^{-3}$

No useful limits on ξ' $[Q_{\tau \to l_R} = \frac{1}{2}(1 - \xi')]$

NEUTRAL CURRENTS

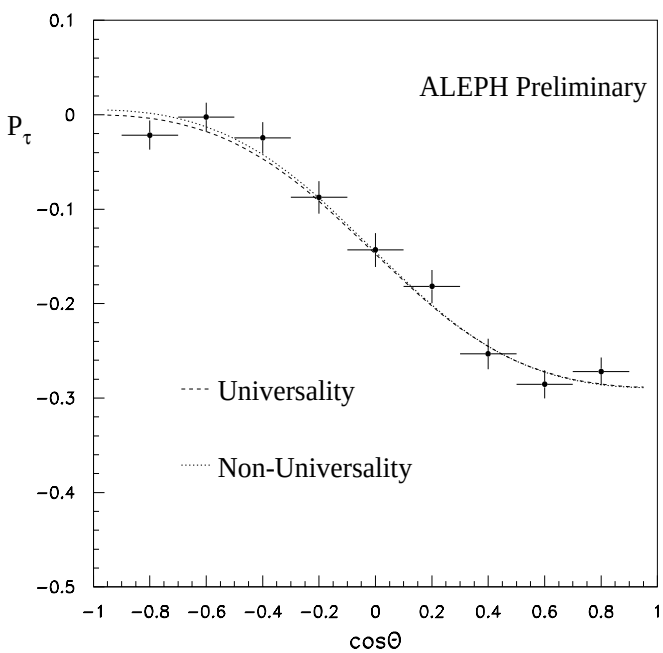
$Z \to l^- l^+$: $\quad \Gamma_l \propto |v_l|^2 + |a_l|^2 \quad ; \quad A_l \equiv \frac{2 v_l a_l}{|v_l|^2 + |a_l|^2}$

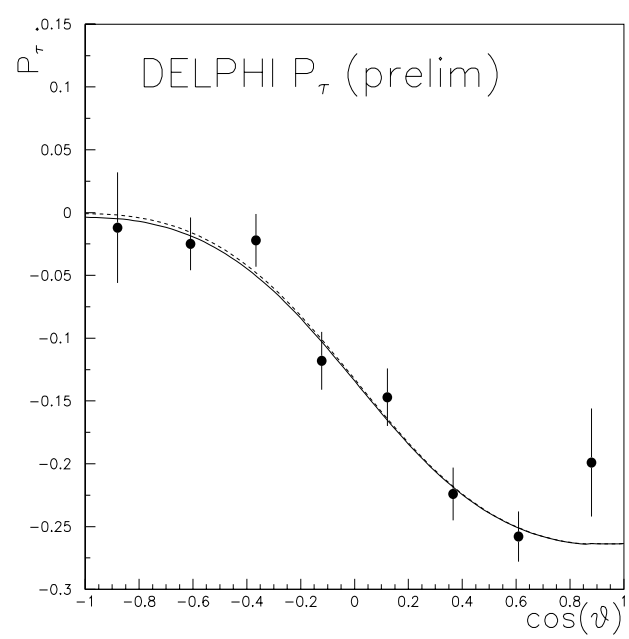
$$\mathcal{A}^l_{FB} = \frac{3}{4} A_e A_l \; ; \; \mathcal{A}^l_{Pol} = -A_l \; ; \; \mathcal{A}^l_{FB,Pol} = -\frac{3}{4} A_e$$

$$\mathcal{A}_{LR} = A_e \quad ; \quad \mathcal{A}^l_{FB,LR} = \frac{3}{4} A_l$$

$\mathcal{A}^l_{Pol}$ and $\mathcal{A}^l_{FB,Pol}$ only available for the τ

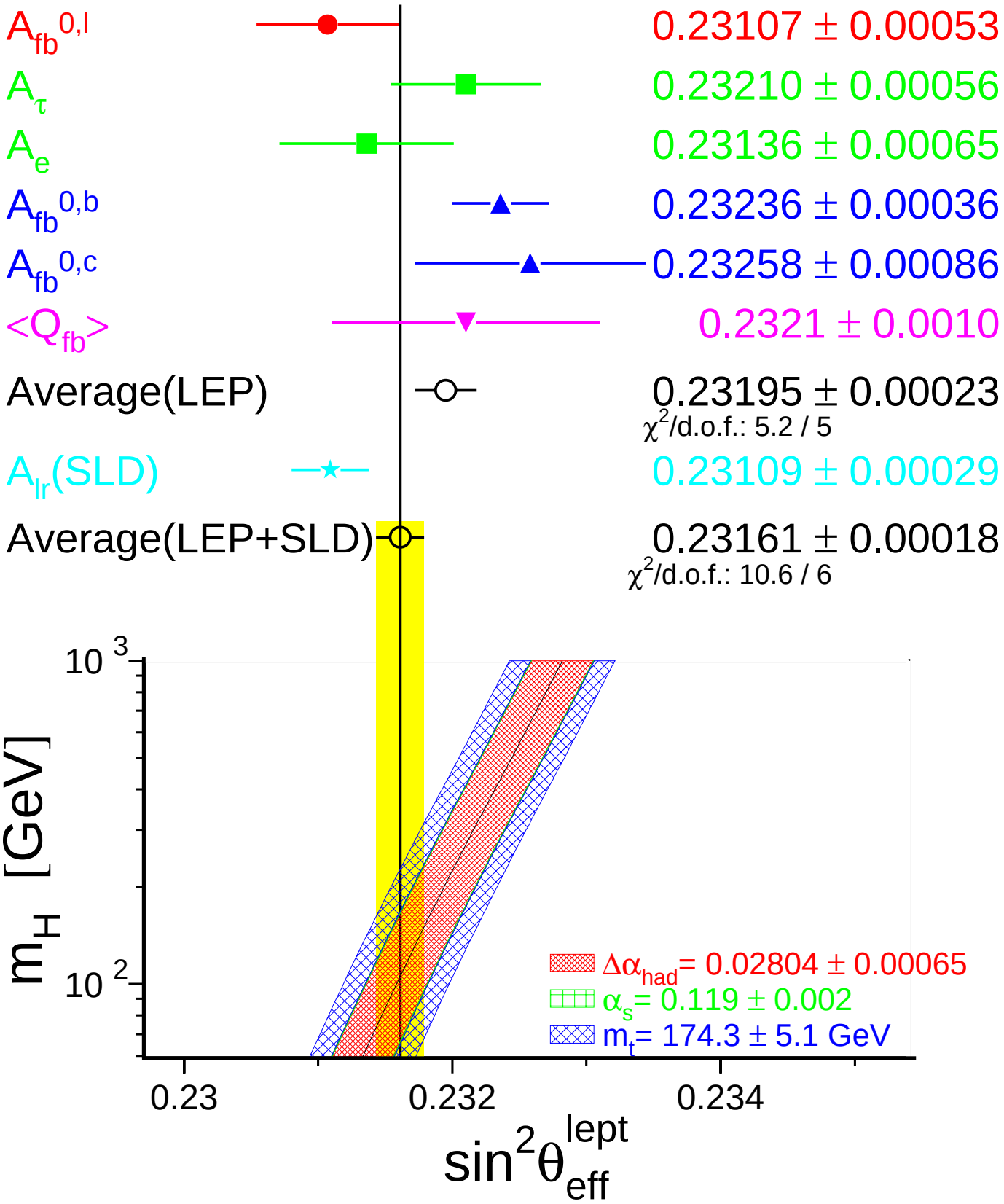
$$\mathcal{P}_\tau(\cos\theta) = -\frac{A_\tau\,(1+\cos^2\theta) + 2\,A_e\,\cos\theta}{(1+\cos^2\theta) + 2\,A_\tau\,A_e\,\cos\theta}$$





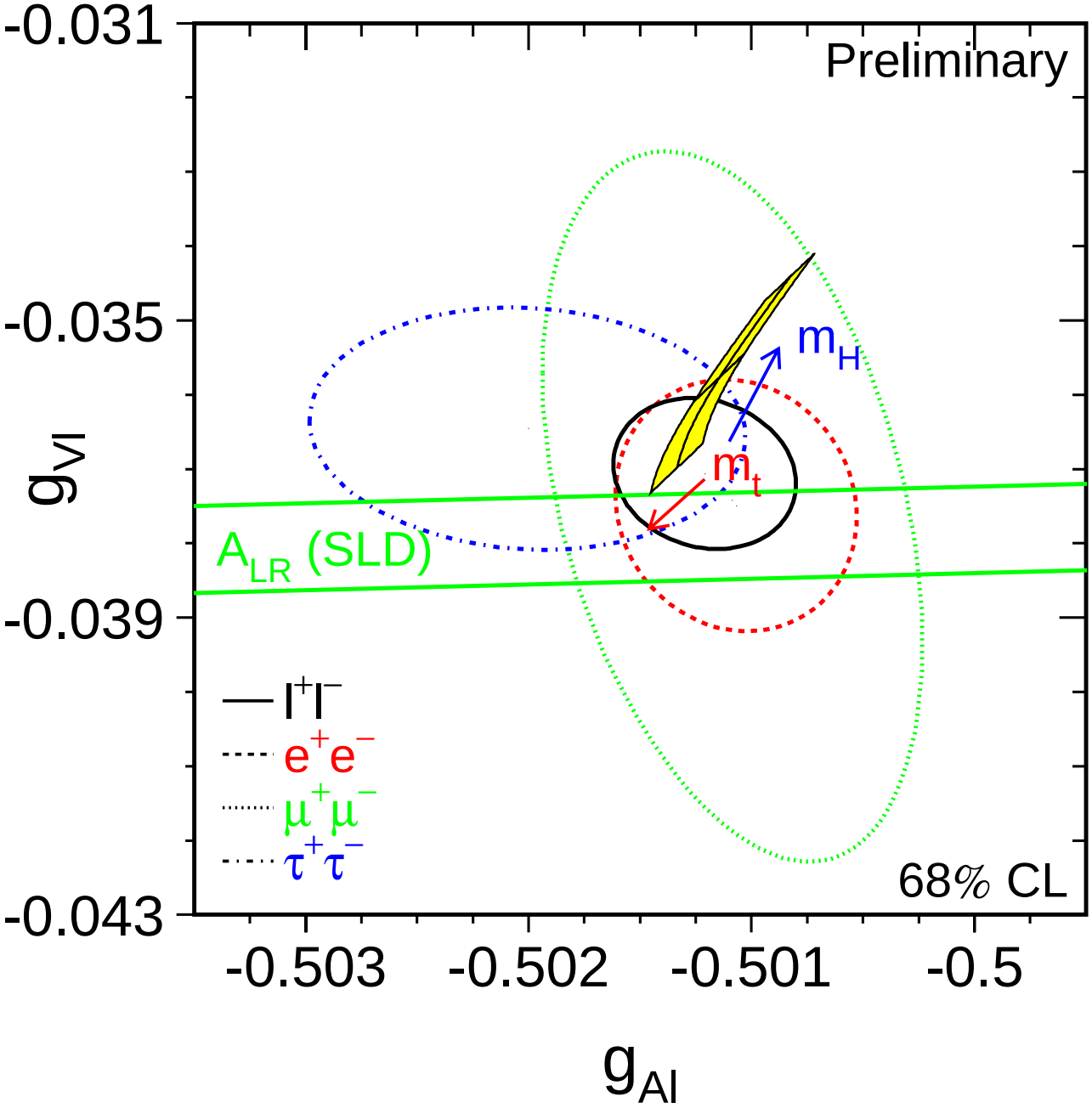
$A_\tau = 0.1425 \pm 0.0044$; $A_e = 0.1483 \pm 0.0051$

Preliminary



NEUTRAL–CURRENT UNIVERSALITY

LEPEWWG



THE ν_τ MASS

m_{ν_τ} from 2D likelihood fits of E_X and M_X distributions in $\tau \to \nu_\tau X$ near the end-point

95% CL Upper Limits (MeV)

X	ALEPH	CLEO	DELPHI	OPAL
3π	25.7		28*	35.3
$3\pi\pi^0$		28		
5π	23.1	30		43.2
Combined	18.2	28*	28*	27.6

If $m_{\nu_\tau}/m_{\nu_e} \sim (m_\tau/m_e)^2$ → $m_{\nu_e} < 1.5$ eV

$g_\tau/g_\mu/g_e$ Universality: (Dova-Swain-Taylor)

$m_{\nu_\tau} < 38$ MeV (95% CL)

SuperKamiokande: *(Assuming $\nu_\mu \to \nu_\tau$, $m_{\nu_\tau} \gg m_{\nu_\mu}$)*

0.02 eV $< m_{\nu_\tau} < 0.08$ eV (90% CL)

LEPTON NUMBER VIOLATION

90% CL Upper Limits on $B(\tau^- \to X^-)$ (CLEO)

X^-	UL, 10^{-6}	X	UL, 10^{-6}
$e^-\gamma$	2.7	$\mu^-\gamma$	3.0
$e^-e^+e^-$	2.9	$\mu^-\mu^+\mu^-$	1.9
$e^-e^+\mu^-$	1.7	$\mu^-\mu^+e^-$	1.8
$e^-\mu^+e^-$	1.5	$\mu^-e^+\mu^-$	1.5
$e^-\pi^0$	3.7	$\mu^-\pi^0$	4.0
$e^-\eta$	8.2	$\mu^-\eta$	9.6
$e^-\rho^0$	2.0	$\mu^-\rho^0$	6.3
e^-K^{*0}	5.1	μ^-K^{*0}	7.5
$e^-\bar{K}^{*0}$	7.4	$\mu^-\bar{K}^{*0}$	7.5
$e^-\phi$	6.9	$\mu^-\phi$	7.0
$e^-\pi^+\pi^-$	2.2	$\mu^-\pi^+\pi^-$	8.2
$e^-\pi^+K^-$	6.4	$\mu^-\pi^+K^-$	7.5
$e^-K^+\pi^-$	3.8	$\mu^-K^+\pi^-$	7.4
$e^-K^+K^-$	6.0	$\mu^-K^+K^-$	15
$e^+\pi^-\pi^-$	1.9	$\mu^+\pi^-\pi^-$	3.4
$e^+\pi^-K^-$	2.1	$\mu^+\pi^-K^-$	7.0
$e^+K^-K^-$	3.8	$\mu^+K^-K^-$	6.0
$e^-\pi^0\pi^0$	6.5	$\mu^-\pi^0\pi^0$	14
$e^-\pi^0\eta$	24	$\mu^-\pi^0\eta$	22
$e^-\eta\eta$	35	$\mu^-\eta\eta$	60

HADRONIC DECAYS

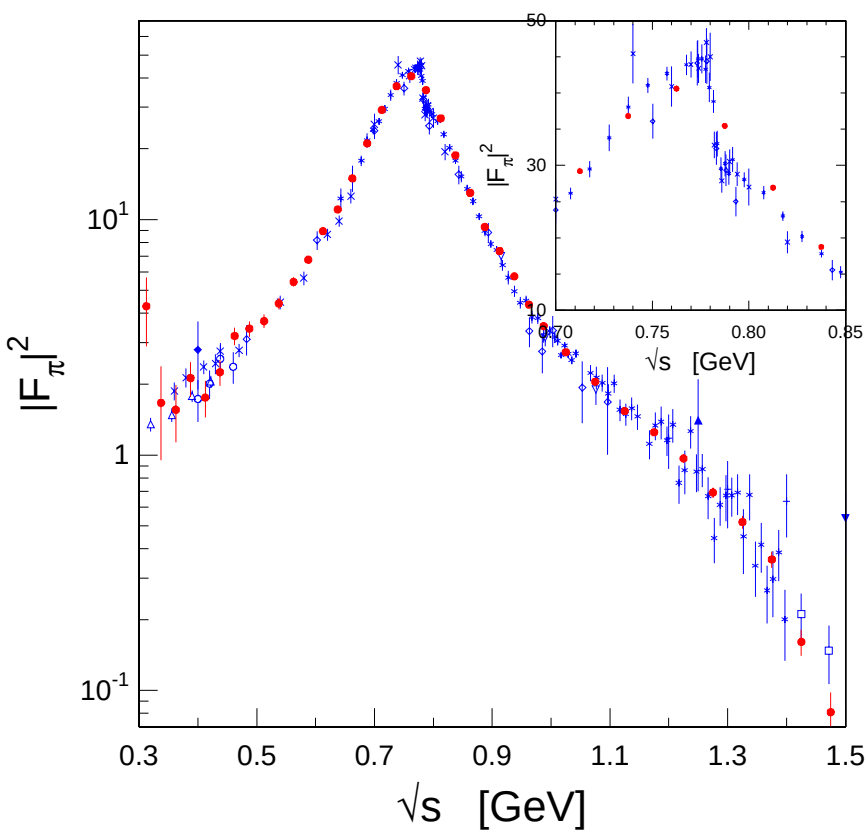
Only lepton massive enough to decay into hadrons

$\tau^- \to \nu_\tau H^-$ probes the hadronic $V - A$ current

$$\langle H^- | \bar{d}_\theta \gamma^\mu (1 - \gamma_5) u | 0 \rangle$$

$\tau^- \to \nu_\tau \pi^- \pi^0$: Pion Form Factor

$$\langle \pi^- \pi^0 | \, \bar{d} \gamma^\mu u \, | 0 \rangle \equiv \sqrt{2} \, F_\pi(s) \, (p_{\pi^-} - p_{\pi^0})^\mu$$



CLEO

(10^5 selected events)

τ data slightly above e^+e^- data

[Br higher than CVC prediction by $(3.2 \pm 1.4)\%$]

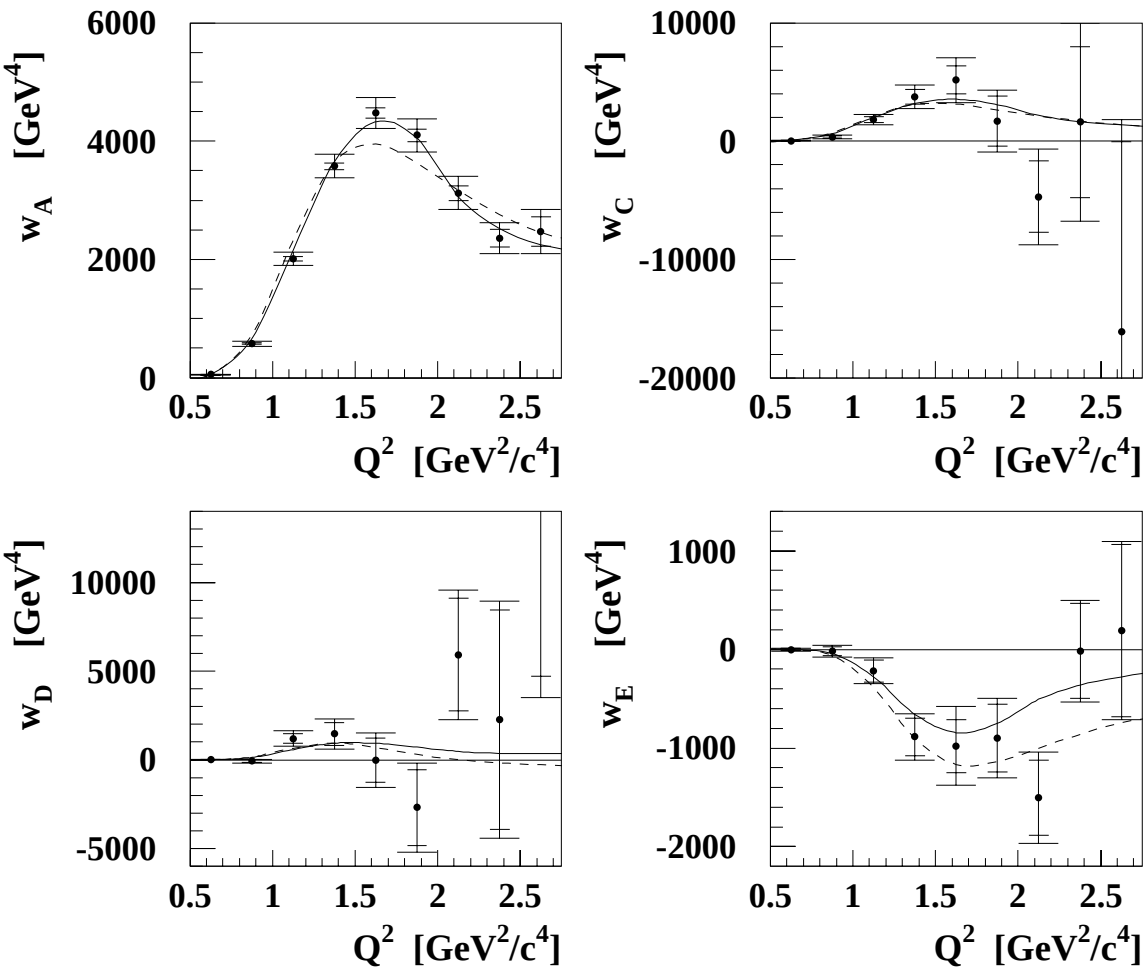
$\tau^- \to \nu_\tau \pi^- 2\pi^0$

4 Form Factors: $\mathcal{F}_{1,2}$ (1^+), $\mathcal{F}_3$ (1^-), $\mathcal{F}_4$ $(0^+, 0^-)$

1^- and 0^+ forbidden by G-Parity ; $\mathcal{F}_4^{0^-} \sim m_\pi$

$\mathcal{F}_{1,2}$ → 4 Structure Functions (Kühn–Mirkes)

CLEO (5×10^4 selected events)



Non Axial Contribution $< 16.6\%$ (95% CL)

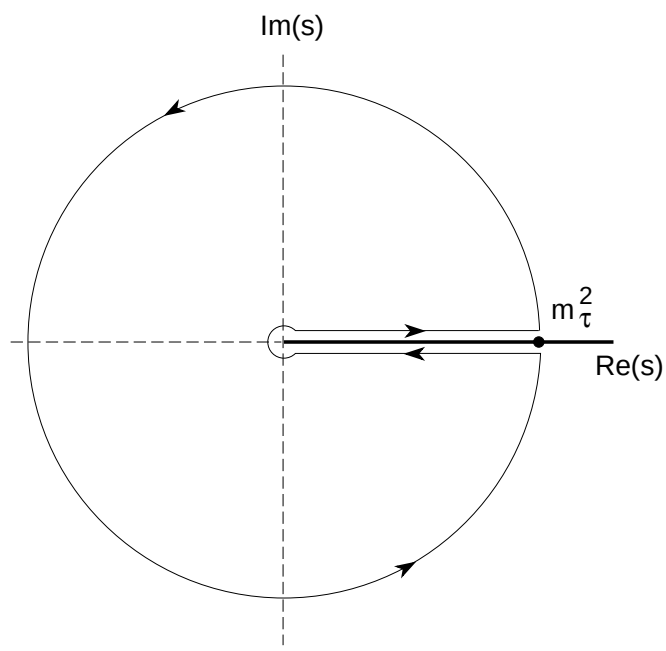
$h_{\nu_\tau} = -1.02 \pm 0.13 \pm 0.03_{model}$ [$|h_{\nu_\tau}| = 1.0000 \pm 0.0057$]

QCD TESTS

$$R_\tau \equiv \frac{\Gamma(\tau \to \nu_\tau + \text{hadrons})}{\Gamma(\tau \to \nu_\tau\, e\, \bar{\nu}_e)} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

The inclusive τ decay width can be accurately predicted in QCD (Braaten–Narison–Pich)

$$\begin{aligned} R_\tau &= 12\,\pi \int_0^1 dx\,(1-x)^2\,\left[(1+2x)\,\mathrm{Im}\,\Pi^T(s) + \mathrm{Im}\,\Pi^L(s)\right] \\ &= 6\,\pi\, i \oint_{|x|=1} dx\,(1-x)^2\,\left[(1+2x)\,\Pi^{T+L}(s) - 2\,x\,\Pi^L(s)\right] \end{aligned}$$



$(x = s/m_\tau^2)$

$$\Pi^J(s) \equiv |V_{ud}^2|\left[\Pi^J_{ud,V}(s) + \Pi^J_{ud,A}(s)\right] + |V_{us}^2|\left[\Pi^J_{us,V}(s) + \Pi^J_{us,A}(s)\right]$$

$$i\int d^4x\, e^{iqx}\,\langle 0|T\left(\mathcal{J}^\mu_{ij}(x)\,\mathcal{J}^\nu_{ij}(0)^\dagger\right)|0\rangle =$$

$$\left(-g^{\mu\nu}q^2 + q^\mu q^\nu\right)\,\Pi^T_{ij,\mathcal{J}}(q^2) + q^\mu q^\nu\,\Pi^L_{ij,\mathcal{J}}(q^2)$$

$$R_\tau = N_C\, S_{EW} \left\{ 1 + \delta'_{EW} + \delta_P + \delta_{NP} \right\}$$

$S_{EW} = 1.0194$; $\delta'_{EW} = 0.0010$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

$$\delta_P = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + \cdots \approx 20\%$$

$$\delta_{NP} = \sum_{n\geq 2} C_{2n}/m_\tau^{2n} \sim C_6/m_\tau^6 < 1\%$$

Similar predictions for $R_{\tau,V}$, $R_{\tau,A}$, $R_{\tau,S}$ and

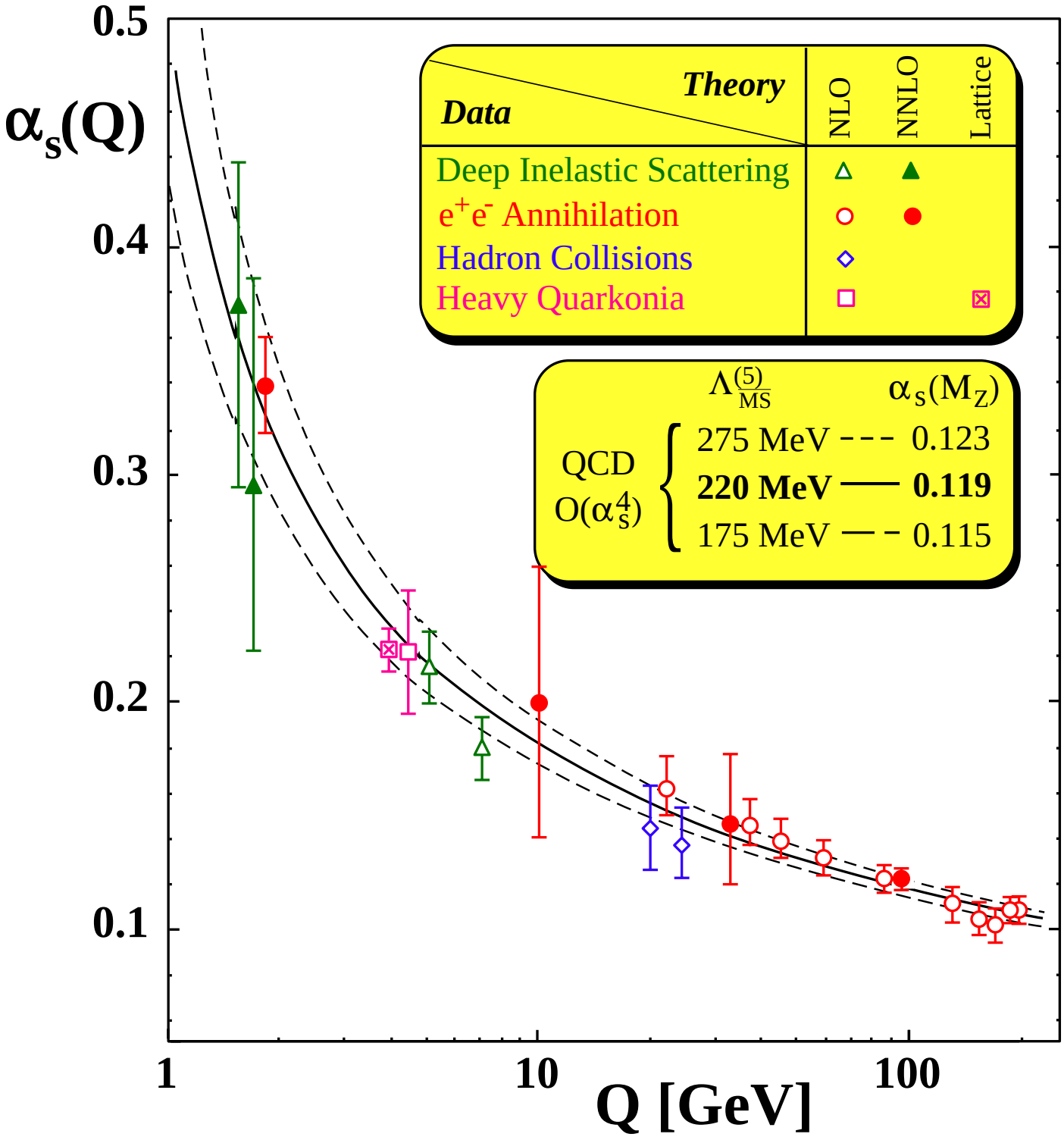
$$R_\tau^{kl} \equiv \int ds \left(1 - s/m_\tau^2\right)^k \left(s/m_\tau^2\right)^l \frac{dR_\tau}{ds}$$

Non-Perturbative contributions fitted from data

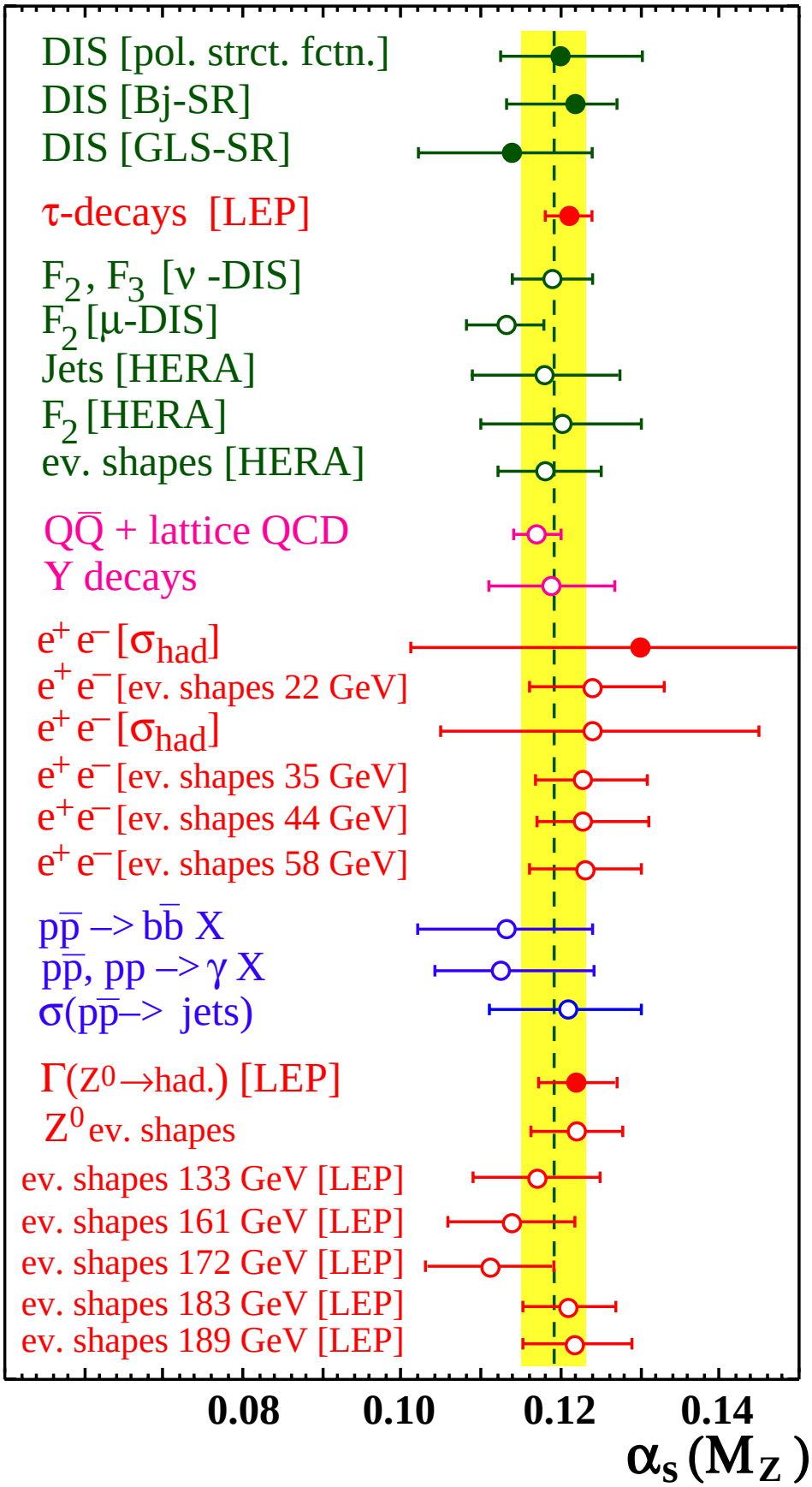
	ALEPH	OPAL
δ_P	0.202 ± 0.013	
δ_{NP}	-0.003 ± 0.004	-0.0024 ± 0.0025
$\alpha_s(m_\tau)$ [CI]	0.345 ± 0.018	0.348 ± 0.021
[FOPT]	0.322 ± 0.020	0.324 ± 0.014
$\alpha_s(M_Z)$ [CI]	0.1212 ± 0.0021	0.1219 ± 0.0020
[FOPT]	0.1186 ± 0.0024	0.1191 ± 0.0015
$R_{\tau,V+A}$	3.492 ± 0.016	3.484 ± 0.024
$R_{\tau,V}$	1.775 ± 0.017	1.764 ± 0.016
$R_{\tau,A}$	1.717 ± 0.018	1.720 ± 0.017
$R_{\tau,S}$	0.155 ± 0.008	

PDG'98: $\alpha_s(M_Z) = 0.119 \pm 0.002$

MEASUREMENTS OF $\alpha_s(Q)$ (S. Bethke)

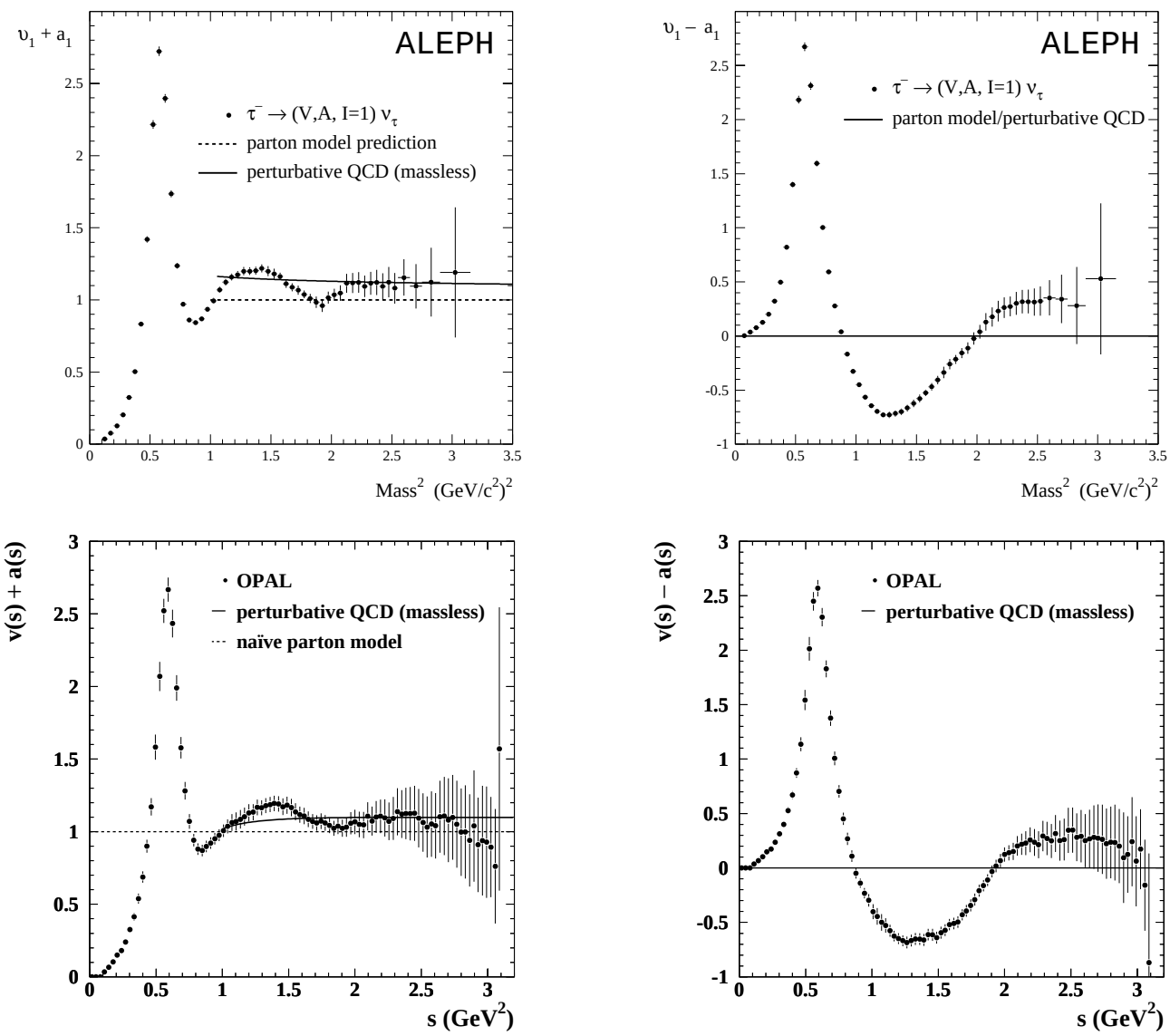


MEASUREMENTS OF $\alpha_s(M_Z)$ (S. Bethke)



SPECTRAL FUNCTIONS

$$v(s) \equiv 2\pi \, \mathrm{Im}\, \Pi^{T+L}_{ud,V}(s) \quad ; \quad a(s) \equiv 2\pi \, \mathrm{Im}\, \Pi^{T+L}_{ud,A}(s)$$



Important Information:

$v(s)$ ➔ CVC , $\alpha^{-1}(M_Z) = 128.933 \pm 0.021$,

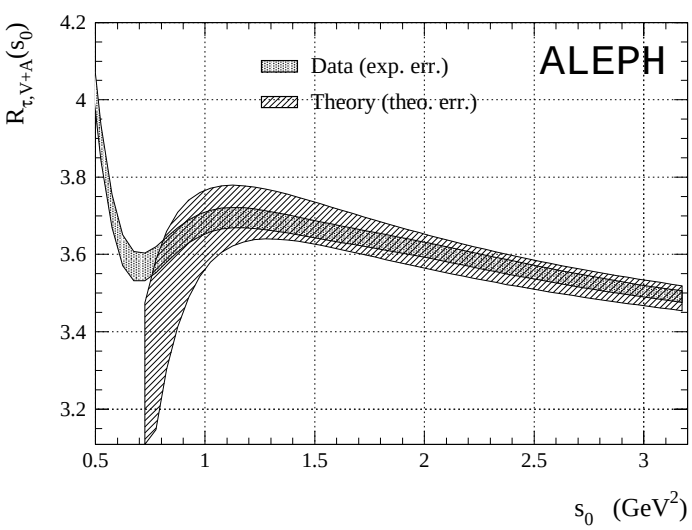
$a_\mu^{had} = (692.4 \pm 6.2) \times 10^{-10}$ (Davier et al), . . .

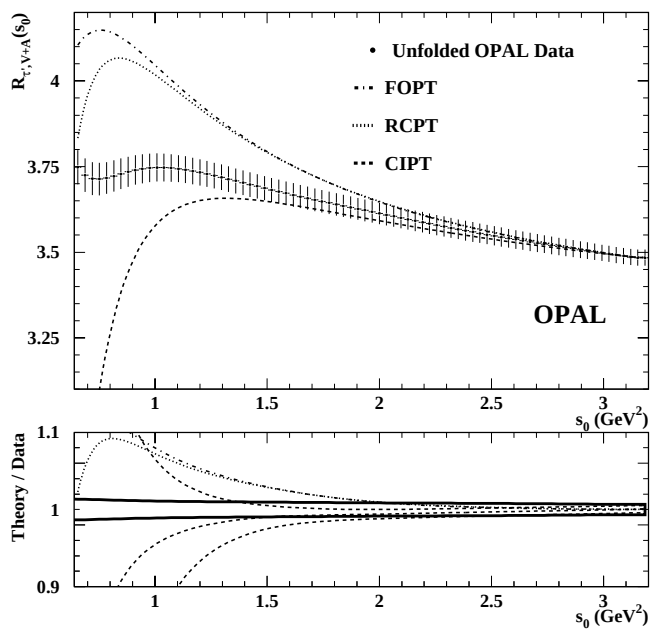
$v(s) - a(s)$ ➔ f_π , $m^2_{\pi^\pm} - m^2_{\pi^0}$, $F_A/\langle r^2_\pi \rangle$, . . .

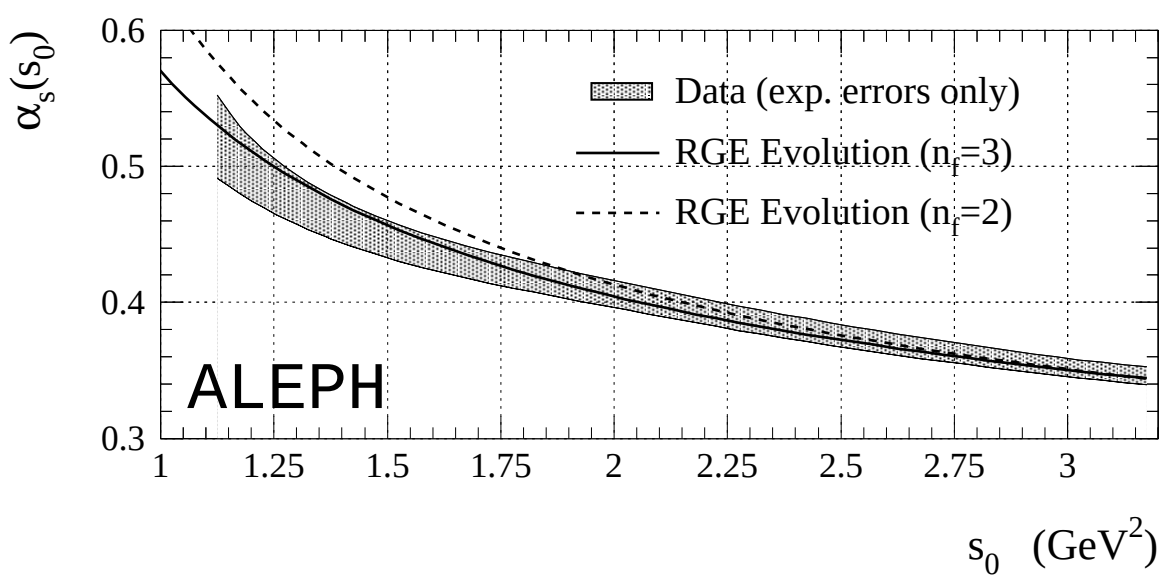
Chiral Sum Rules (SCSB)

SCALE DEPENDENCE $(m_\tau^2 \to s_0)$

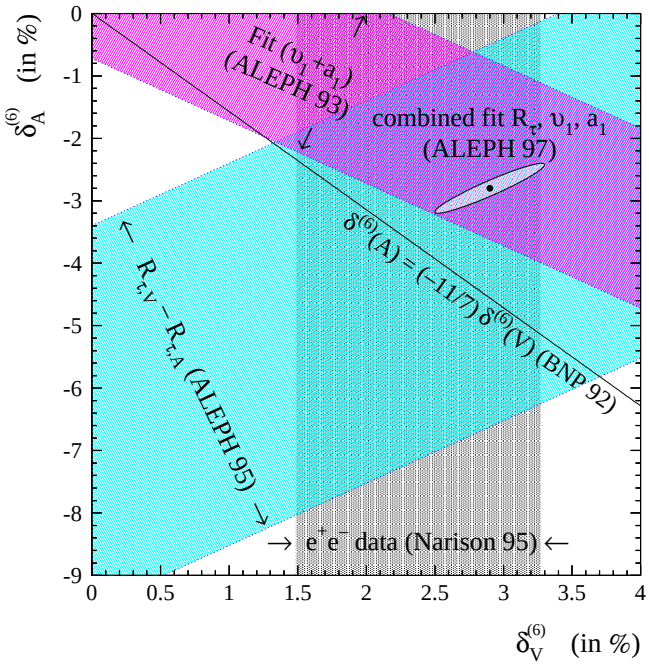
$$R_\tau(s_0) \equiv 12\pi\, S_{EW} \int_0^{s_0} \frac{ds}{s_0} \left(1 - \frac{s}{s_0}\right)^2 \times \left[\left(1 + 2\frac{s}{s_0}\right) \mathrm{Im}\Pi^T(s) + \mathrm{Im}\Pi^L(s)\right]$$

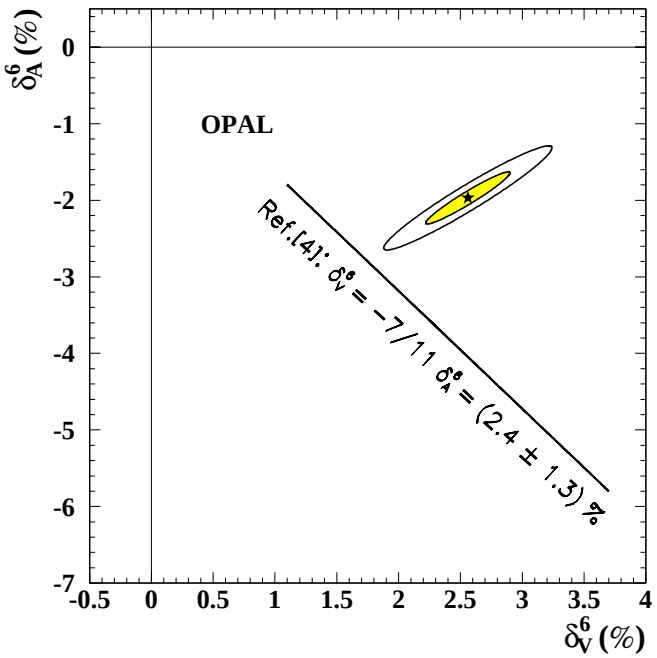


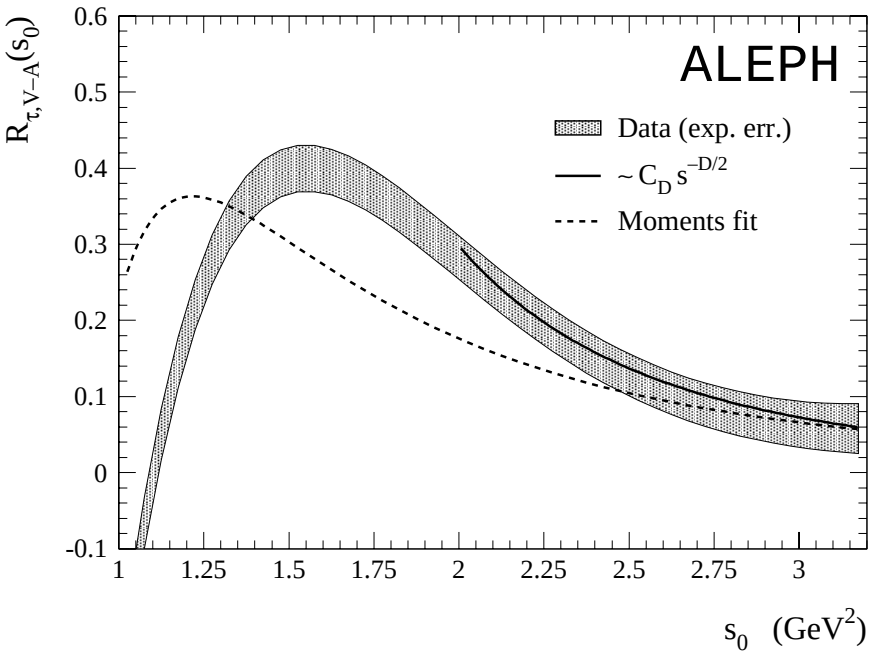




$D = 6$ CONTRIBUTION TO $R_{\tau,V/A}$







Fit to $R_{\tau,V-A}(s_0) = \frac{3}{2}\,|V_{ud}|^2\, S_{EW} \left(\frac{C_D}{s_0^{D/2}}\right)$

$\Rightarrow$ $D = 6.9 \pm 0.9$

In agreement with OPE prediction $(D = 6)$

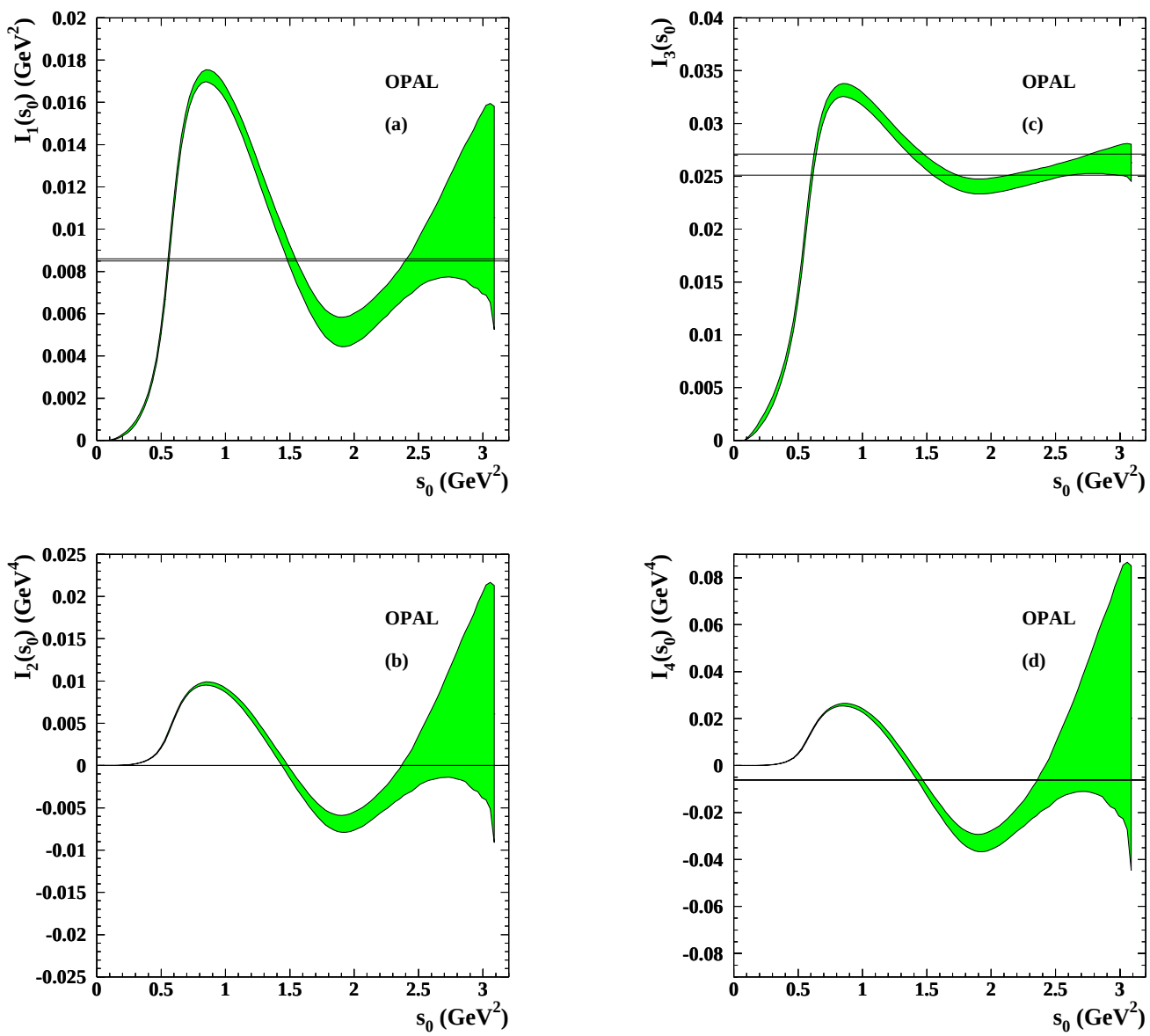
CHIRAL SUM RULES

$$\rho(s) \equiv \frac{1}{2\pi}\left\{\mathrm{Im}\,\Pi^{T+L}_{ud,V}(s) - \mathrm{Im}\,\Pi^{T+L}_{ud,A}(s)\right\}$$

When $s_0 \to \infty$

$$I_1 \equiv \int_0^{s_0} \rho(s) = f_\pi^2 \quad ; \quad I_3 \equiv \int_0^{s_0} \frac{ds}{s}\,\rho(s) = f_\pi^2\,\frac{\langle r_\pi^2\rangle}{3} - F_A$$

$$I_2 \equiv \int_0^{s_0} s\,\rho(s) = 0 \quad ; \quad I_4 \equiv \int_0^{s_0} ds\, s\,\ln(s)\,\rho(s) = -\frac{4\pi f_\pi^2}{3\alpha}\left(m^2_{\pi^\pm} - m^2_{\pi^0}\right)$$



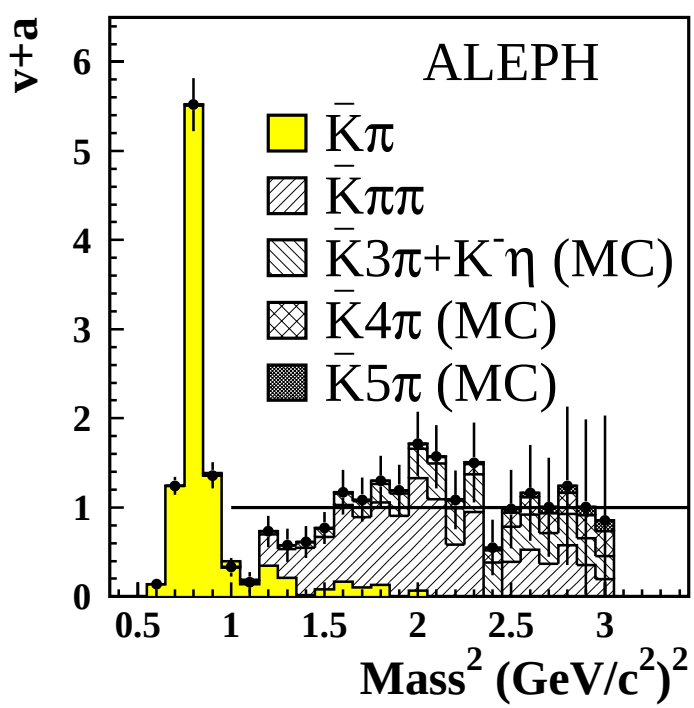
KAONS IN τ DECAY

Many results (ALEPH, CLEO, DELPHI, OPAL)

$\mathrm{Br}(\tau^- \to \nu_\tau X^-)$ World Averages

X^-	Br (%)	X^-	Br (%)
K^-	0.690 ± 0.025	$(K\pi)^-$	1.30 ± 0.07
$\bar{K}^0\pi^-\pi^0$	0.356 ± 0.041	$K^-\pi^+\pi^-$	0.310 ± 0.058
$K^-\pi^+\pi^-\pi^0$	0.070 ± 0.025	K^0K^-	0.159 ± 0.017
$K^-K^+\pi^-$	0.156 ± 0.018	$K^0_SK^0_S\pi^-$	0.024 ± 0.005
$K^0K^-\pi^0$	0.148 ± 0.021	$K^-K^+\pi^-\pi^0$	0.044 ± 0.018

Strange Spectral Function (ALEPH)



→ Moments

$$R^{kl}_{\tau,S} \equiv \int ds\, w_{kl}(s)\, \frac{dR_{\tau,S}}{ds}$$

$$w_{kl}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l$$

$R_{\tau,S}$ ➔ m_s (Prades–Pich)

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = 3 \sum_D \left[\delta_{ud}^{kl\,(D)} - \delta_{us}^{kl\,(D)} \right]$$

$$\approx 24 \frac{m_s^2(m_\tau)}{m_\tau^2} \Delta_{kl}(a_\tau) - 48\,\pi^2 \frac{\delta O_4}{m_\tau^4} Q_{kl}(a_\tau)$$

$$\delta O_4 \equiv \langle 0 | m_s \bar{s}s - m_d \bar{d}d | 0 \rangle \approx f_\pi^2 \left(m_{\pi^\pm}^2 - m_{K^\pm}^2 \right)$$

$\Delta_{00}(a_\tau) = 2.0 \pm 0.5$ (Bad perturbative behaviour)

(k,l)	δR_τ^{kl} (ALEPH)	$m_s(m_\tau)$ (MeV)
$(0,0)$	0.394 ± 0.137	$143 \pm 31_{exp} \pm 18_{th}$
$(1,0)$	0.383 ± 0.078	$121 \pm 17_{exp} \pm 18_{th}$
$(2,0)$	0.373 ± 0.054	$106 \pm 12_{exp} \pm 21_{th}$

➔ $m_s(m_\tau) = (119 \pm 12_{exp} \pm 18_{th} \pm 10_{V_{us}})$ MeV

$m_s(1\,\text{GeV}) = 164 \pm 31$; $m_s(2\,\text{GeV}) = 114 \pm 23$

Subtracting the known K/π poles (J=0) one gets an upper (lower) bound on Im $\Pi^{L+T}(s)$ [Im $\Pi^L(s)$]

➔ $m_s(m_\tau) < 202$ MeV (in agreement with ALEPH)

SUMMARY

The τ is an ideal tool to test the Standard Model

- Lepton Universality tested to rather good accuracy

- $V-A$ Structure verified in $\mu \to e\,\bar{\nu}_e\,\nu_\mu$, but not yet in $\tau \to l\,\bar{\nu}_l\,\nu_\tau$. Good limits on $\tau_R \to l\,\bar{\nu}_l\,\nu_\tau$

- Wonderful QCD Laboratory to study the hadronic V, A currents
  - Exclusive: Chiral Dynamics, Resonances, . . .
  - Inclusive: α_s , m_s , $\langle 0|G^2|0\rangle$, . . .

- New Physics could also show up ($\not{L}$, CP, m_{ν_τ}, a_τ, $d_\tau^{\gamma,Z}$, . . .)

A remarkable progress has been already achieved

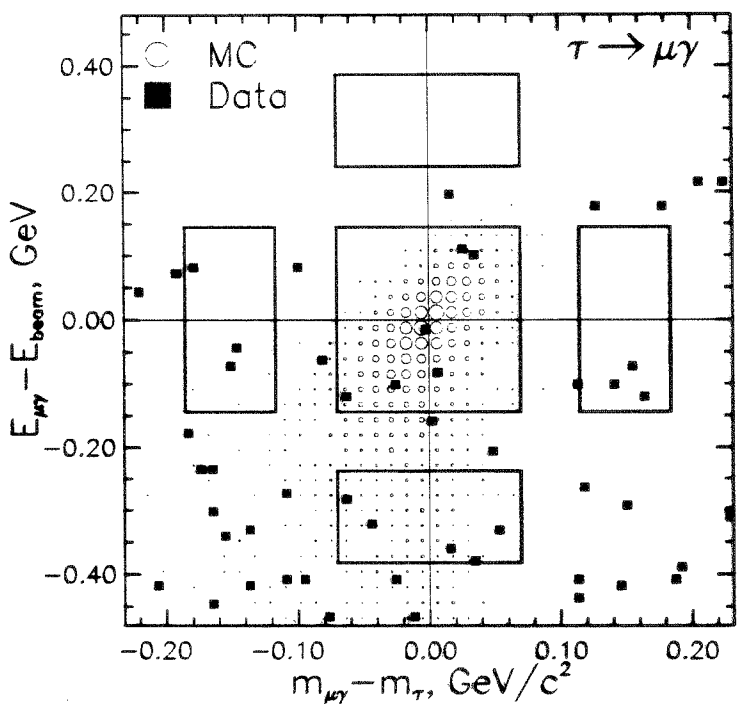
Large room for future improvements

Search for $\tau \to \mu\gamma$
(CLEO Collaboration)

Lepton-Photon, August 1999
All quoted results are preliminary!

CLEO, Phys. Rev. D 55 (1997): 4.24×10^6 $\tau^+\tau^-$ pairs; $\mathcal{B}(\tau \to \mu\gamma) < 3.0 \times 10^{-6}$ at 90% CL

this search: $13.8\,\mathrm{fb}^{-1} \Longrightarrow 12.6 \times 10^6$ $\tau^+\tau^-$ pairs (full CLEO II)



	Meth. of prev. search	Unbinned EML fit
Number of signal events	$n_0 = 6$	$s = 1.8$
Expected backgr. rate, events	5.5 ± 0.5	-
Statistical significance	-	1.0σ
UL at 90% CL, events	5.8	3.8
UL for $\mathcal{B}(\tau \to \mu\gamma)$ at 90% CL	1.8×10^{-6}	$\mathbf{1.0 \times 10^{-6}}$

restricts the parameter space for some versions of MSSM

Summarizing the Left-right
Forward-Backward Asymmetry